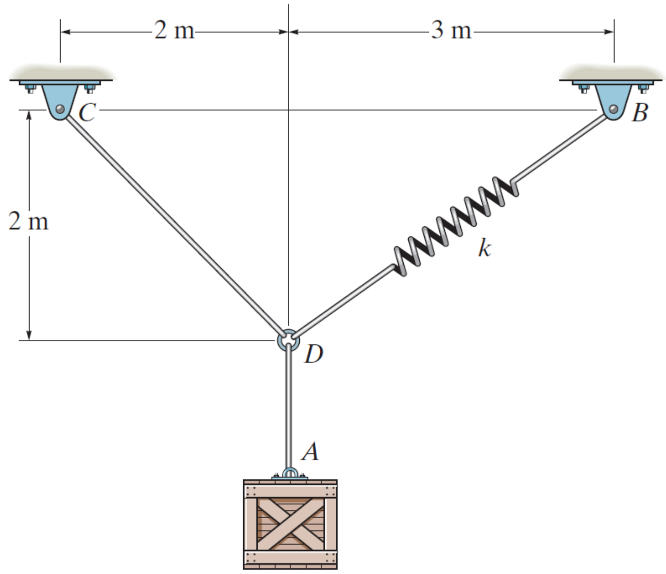
Problem 3-18

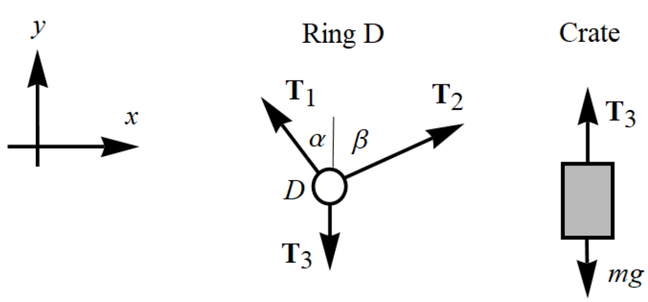
If the spring DB has an unstretched length of 2 m, determine the stiffness of the spring to hold the 40-kg crate in the position shown.



Probs. 3–18/19

Solution

Draw one free-body diagram for the ring at D and one free-body diagram for the crate.



In order for the system to be in equilibrium, the sum of the forces in each direction must be zero.

$$\sum F_x = 0: \qquad T_2 \sin\beta - T_1 \sin\alpha = 0 \qquad 0 = 0$$

$$\sum F_y = 0: \qquad T_1 \cos\alpha + T_2 \cos\beta - T_3 = 0 \qquad T_3 - mg = 0$$

Since $T_3 = mg$ and $T_2 = k(\sqrt{2^2+3^2}\text{ m} - 2\text{ m})$, the system of equations reduces to

$$k\left(\sqrt{2^2+3^2} - 2\right)\sin\beta - T_1 \sin\alpha = 0 \tag{1}$$

$$T_1 \cos\alpha + k\left(\sqrt{2^2+3^2} - 2\right)\cos\beta - mg = 0. \tag{2}$$

Solve equation (1) for T_1

$$T_1 = \frac{k\left(\sqrt{2^2+3^2}-2\right)\sin\beta}{\sin\alpha}$$

and substitute it into equation (2). Solve for k.

$$\frac{k\left(\sqrt{2^2+3^2}-2\right)\sin\beta}{\sin\alpha}\cos\alpha + k\left(\sqrt{2^2+3^2}-2\right)\cos\beta - mg = 0$$

$$k\left(\sqrt{2^2+3^2}-2\right)\sin\beta\cot\alpha + k\left(\sqrt{2^2+3^2}-2\right)\cos\beta = mg$$

$$k = \frac{mg}{\left(\sqrt{2^2+3^2}-2\right)\sin\beta\cot\alpha + \left(\sqrt{2^2+3^2}-2\right)\cos\beta}$$

Use trigonometry to determine α and β.

$$\tan\alpha = \frac{2}{2} \quad\rightarrow\quad \alpha = \tan^{-1}(1) = 45^\circ$$

$$\tan\beta = \frac{3}{2} \quad\rightarrow\quad \beta = \tan^{-1}\left(\frac{3}{2}\right) \approx 56.3^\circ$$

Therefore, since $m = 40$ kg and $g = 9.81$ m/s^2, the spring stiffness is

$$k \approx 176\ \frac{\text{N}}{\text{m}}.$$