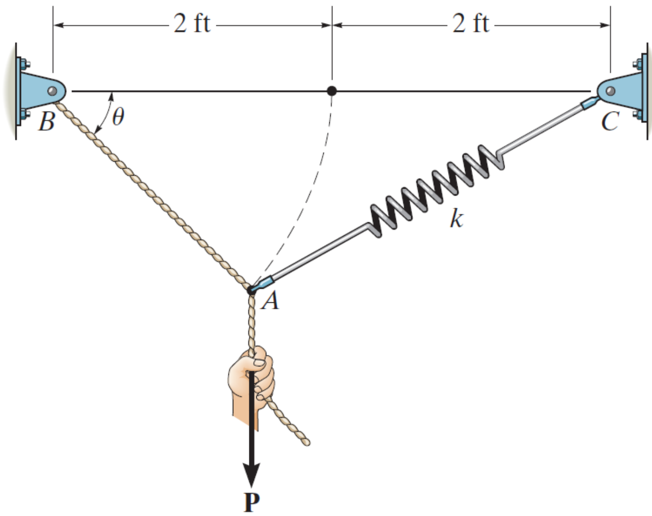
Problem 3-21

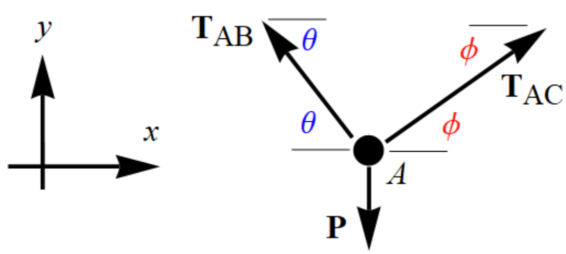
Determine the unstretched length of spring AC if a force $P = 80$ lb causes the angle $\theta = 60°$ for equilibrium. Cord AB is 2 ft long. Take $k = 50$ lb/ft.



Probs. 3–20/21

Solution

Draw a free-body diagram for the point at A.



In order for the system to be in equilibrium, the sum of the forces in each direction must be zero.

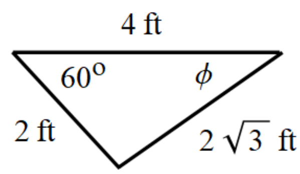
$$\sum F_x = 0: \qquad T_{AC}\cos\phi - T_{AB}\cos\theta = 0$$

$$\sum F_y = 0: \qquad T_{AC}\sin\phi + T_{AB}\sin\theta - P = 0$$

Since $P = 80$ lb and $\theta = 60°$ and $T_{AC} = k\Delta x_{AC} = (50 \text{ lb/ft})\left[\sqrt{2^2 + 4^2 - 2(2)(4)\cos\theta} \text{ ft} - x_0\right]$, the system of equations reduces to

$$50(2\sqrt{3} - x_0)\cos\phi - T_{AB}\cos 60° = 0 \tag{1}$$

$$50(2\sqrt{3} - x_0)\sin\phi + T_{AB}\sin 60° - 80 = 0. \tag{2}$$



Use the law of sines to determine ϕ.

$$\frac{2\sqrt{3}\text{ ft}}{\sin 60^\circ} = \frac{2\text{ ft}}{\sin\phi} \quad \rightarrow \quad \sin\phi = \frac{1}{\sqrt{3}}\sin 60^\circ \quad \rightarrow \quad \phi = 30^\circ$$

Equations (1) and (2) then reduce further to

$$50(2\sqrt{3} - x_0)\cos 30^\circ - T_{AB}\cos 60^\circ = 0 \tag{3}$$

$$50(2\sqrt{3} - x_0)\sin 30^\circ + T_{AB}\sin 60^\circ - 80 = 0. \tag{4}$$

Solve equation (3) for T_{AB}

$$T_{AB} = \frac{50(2\sqrt{3} - x_0)\cos 30^\circ}{\cos 60^\circ}$$

and plug it into equation (4).

$$50(2\sqrt{3} - x_0)\sin 30^\circ + \left[\frac{50(2\sqrt{3} - x_0)\cos 30^\circ}{\cos 60^\circ}\right]\sin 60^\circ - 80 = 0$$

Solve for x_0.

$$50(2\sqrt{3} - x_0)\left(\sin 30^\circ + \frac{\cos 30^\circ}{\cos 60^\circ}\sin 60^\circ\right) = 80$$

$$50(2\sqrt{3} - x_0)\left(\frac{\sin 30^\circ\cos 60^\circ + \cos 30^\circ\sin 60^\circ}{\cos 60^\circ}\right) = 80$$

$$50(2\sqrt{3} - x_0)\left(\frac{1}{\cos 60^\circ}\right) = 80$$

$$2\sqrt{3} - x_0 = \frac{8}{5}\cos 60^\circ$$

$$x_0 = 2\sqrt{3} - \frac{8}{5}\cos 60^\circ \text{ ft} \approx 2.66 \text{ ft}$$