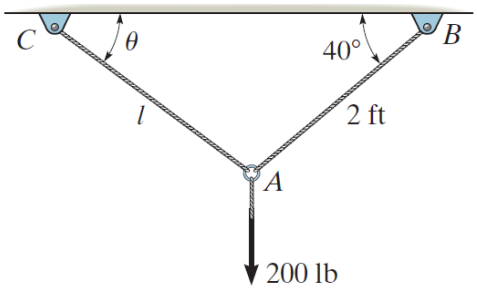
Problem 3-35

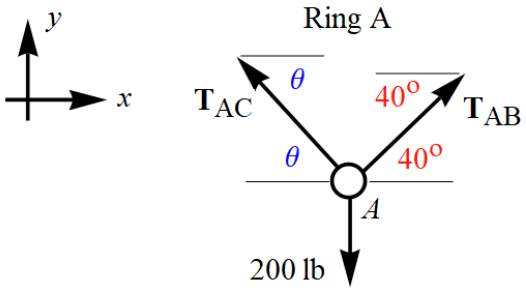
The ring of negligible size is subjected to a vertical force of 200 lb. Determine the required length l of cord AC such that the tension acting in AC is 160 lb. Also, what is the force in cord AB? *Hint:* Use the equilibrium condition to determine the required angle θ for attachment, then determine l using trigonometry applied to triangle ABC.



Prob. 3–35

Solution

Draw one free-body diagram for ring A.



In order for the system to be in equilibrium, the sum of the forces in each direction must be zero.

$$\sum F_x = 0: \qquad T_{\mathrm{AB}} \cos 40^\circ - T_{\mathrm{AC}} \cos\theta = 0$$

$$\sum F_y = 0: \qquad T_{\mathrm{AB}} \sin 40^\circ + T_{\mathrm{AC}} \sin\theta - 200 = 0$$

Set $T_{\mathrm{AC}} = 160$ lb and solve for the terms with θ.

$$160 \cos\theta = T_{\mathrm{AB}} \cos 40^\circ \tag{1}$$

$$160 \sin\theta = 200 - T_{\mathrm{AB}} \sin 40^\circ \tag{2}$$

Square both sides and add the respective sides together to eliminate θ.

$$160^2(\cos^2\theta + \sin^2\theta) = (T_{\mathrm{AB}} \cos 40^\circ)^2 + (200 - T_{\mathrm{AB}} \sin 40^\circ)^2$$

$$160^2(1) = (T_{\mathrm{AB}} \cos 40^\circ)^2 + (200 - T_{\mathrm{AB}} \sin 40^\circ)^2$$

$$160^2 = T_{\mathrm{AB}}^2 \cos^2 40^\circ + 200^2 - 400 T_{\mathrm{AB}} \sin 40^\circ + T_{\mathrm{AB}}^2 \sin^2 40^\circ$$

Solve for T_{AB}.

$$T_{\text{AB}}^2 - 400T_{\text{AB}}\sin 40^\circ + (200^2 - 160^2) = 0$$

$$T_{\text{AB}} = \frac{400\sin 40^\circ \pm \sqrt{400^2\sin^2 40^\circ - 4(200^2 - 160^2)}}{2}$$

$$T_{\text{AB}} \approx \{82.4, 175\} \text{ lb}$$

Both of these are legitimate tensions. Plug each of these into equation (1) to determine the corresponding values of θ.

$$T_{\text{AB}} \approx 82.4: \quad \cos\theta = \frac{T_{\text{AB}}\cos 40^\circ}{160} \quad \rightarrow \quad \theta \approx 66.8^\circ$$

$$T_{\text{AB}} \approx 175: \quad \cos\theta = \frac{T_{\text{AB}}\cos 40^\circ}{160} \quad \rightarrow \quad \theta \approx 33.2^\circ$$

If $T_{\text{AB}} \approx 82.4$ lb, then $\theta \approx 66.8^\circ$ and the law of sines yields

$$\frac{2 \text{ ft}}{\sin\theta} = \frac{l}{\sin 40^\circ} \quad \rightarrow \quad l = \frac{2 \text{ ft}}{\sin\theta}\sin 40^\circ \approx 1.40 \text{ ft}.$$

If $T_{\text{AB}} \approx 175$ lb, then $\theta \approx 33.2^\circ$ and the law of sines yields

$$\frac{2 \text{ ft}}{\sin\theta} = \frac{l}{\sin 40^\circ} \quad \rightarrow \quad l = \frac{2 \text{ ft}}{\sin\theta}\sin 40^\circ \approx 2.34 \text{ ft}.$$