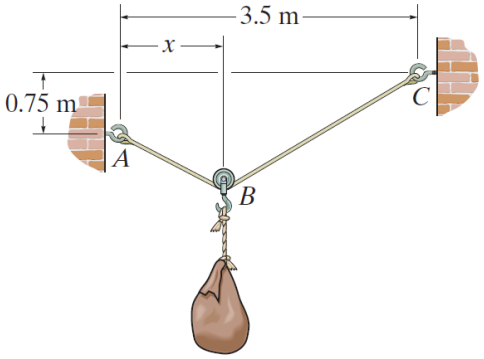
Problem 3-36

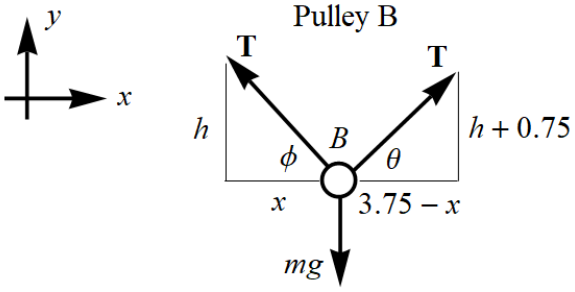
Cable ABC has a length of 5 m. Determine the position x and the tension developed in ABC required for equilibrium of the 100-kg sack. Neglect the size of the pulley at B.



Prob. 3–36

Solution

Draw one free-body diagram for ring A. Note that because the pulley is assumed to be frictionless, the tension is the same in all parts of the cable.

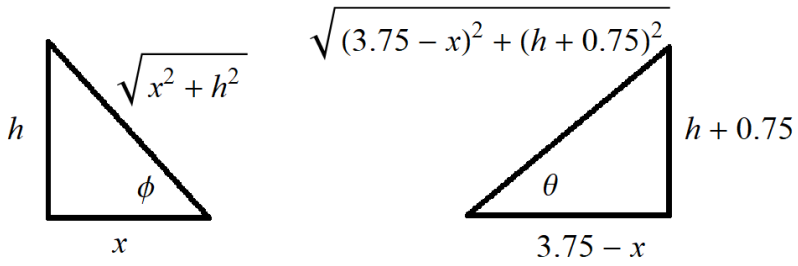


In order for the system to be in equilibrium, the sum of the forces in each direction must be zero.

$$\sum F_x = 0: \qquad T\cos\theta - T\cos\phi = 0 \tag{1}$$

$$\sum F_y = 0: \qquad T\sin\theta + T\sin\phi - mg = 0 \tag{2}$$

Use the Pythagorean theorem to determine each triangle's hypotenuse.



As a result,

$$\cos\phi = \frac{x}{\sqrt{x^2+h^2}} \quad \text{and} \quad \sin\phi = \frac{h}{\sqrt{x^2+h^2}}$$

$$\text{and} \quad \cos\theta = \frac{3.75-x}{\sqrt{(3.75-x)^2+(h+0.75)^2}} \quad \text{and} \quad \sin\theta = \frac{h+0.75}{\sqrt{(3.75-x)^2+(h+0.75)^2}},$$

and equations (1) and (2) become

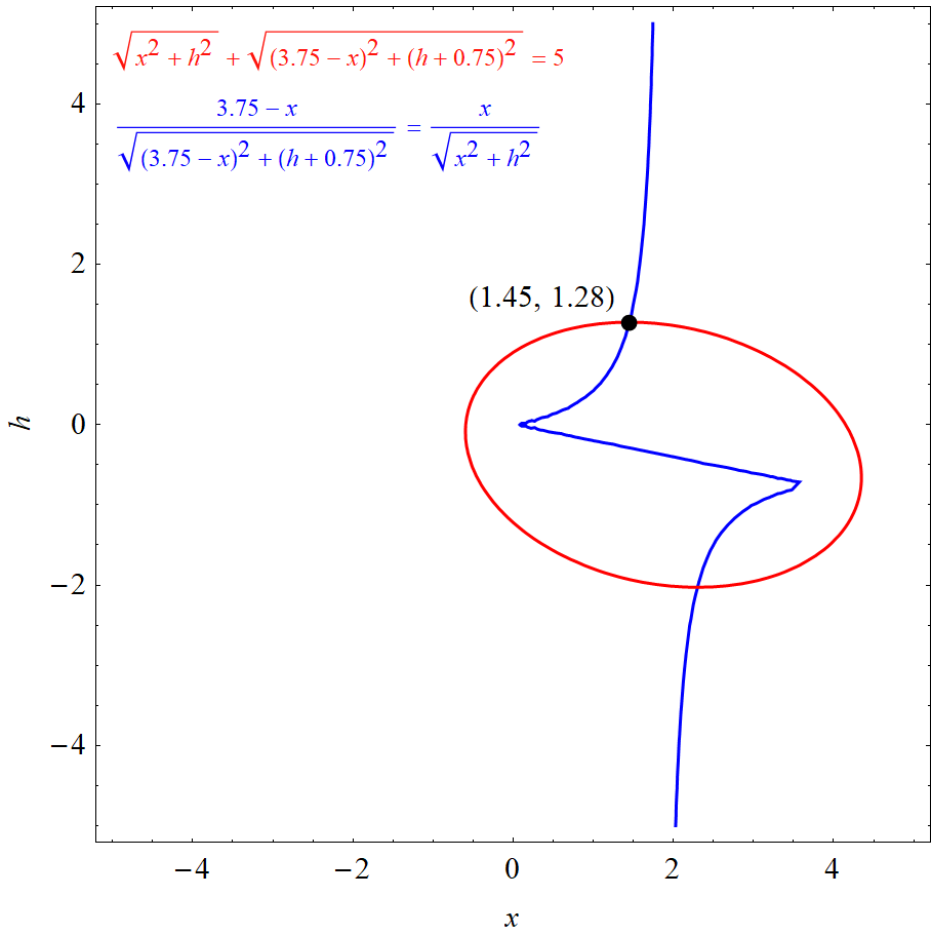
$$\cos\theta = \cos\phi: \quad \frac{3.75-x}{\sqrt{(3.75-x)^2+(h+0.75)^2}} = \frac{x}{\sqrt{x^2+h^2}} \tag{3}$$

$$T = \frac{mg}{\sin\theta+\sin\phi}: \quad T = \frac{100(9.81)}{\frac{h+0.75}{\sqrt{(3.75-x)^2+(h+0.75)^2}}+\frac{h}{\sqrt{x^2+h^2}}}. \tag{4}$$

Use the fact that the length of cable ABC is 5 meters long to get a third equation.

$$\sqrt{x^2+h^2}+\sqrt{(3.75-x)^2+(h+0.75)^2} = 5 \tag{5}$$

Plot the curves in equations (3) and (5) to find the point of intersection.



Only the point with positive x and h is relevant here: $x \approx 1.45$ m and $h \approx 1.28$ m.

Plug these values into equation (4) to get the tension.

$$T = \frac{100(9.81)}{\frac{h+0.75}{\sqrt{(3.75-x)^2+(h+0.75)^2}} + \frac{h}{\sqrt{x^2+h^2}}} \approx 742 \text{ N}$$