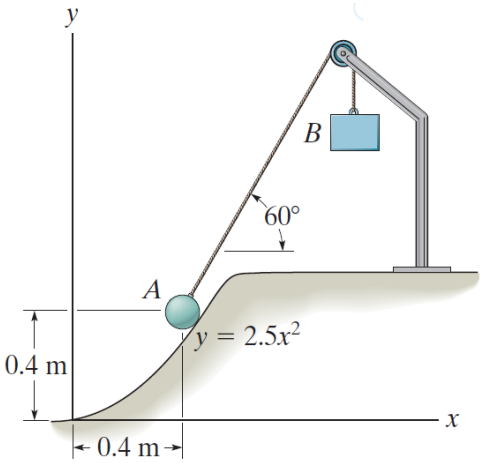
Problem 3-37

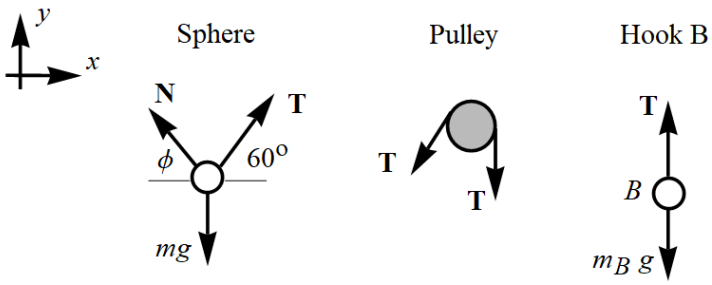
A 4-kg sphere rests on the smooth parabolic surface. Determine the normal force it exerts on the surface and the mass m_B of block B needed to hold it in the equilibrium position shown.



Prob. 3–37

Solution

Draw free-body diagrams for the sphere, the pulley, and hook B.



Note that because the pulley is assumed to be frictionless, the tension is the same in each part of the cable. This means that $T = m_B g$. In order for the system to be in equilibrium, the sum of the forces in each direction must be zero.

$$\sum F_x = 0: \qquad T\cos 60° - N\cos\phi = 0 \tag{1}$$

$$\sum F_y = 0: \qquad T\sin 60° + N\sin\phi - mg = 0 \tag{2}$$

To find ϕ, the angle that **N** makes with the horizontal, start by finding the slope where the sphere makes contact.

$$y' = \frac{d}{dx}(2.5x^2) = 5x$$

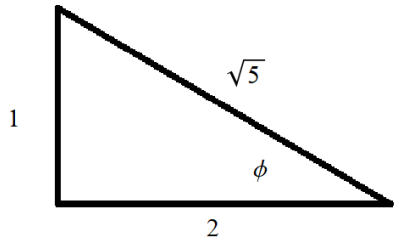
Evaluate it at $x = 0.4$ m.

$$y'(0.4) = 5(0.4) = 2$$

The direction of the normal force is obtained by taking the negative reciprocal.

$$m_\perp = \tan\phi = -\frac{1}{2}$$

Draw the implied triangle in order to determine the sine and cosine.



As a result,

$$\cos\phi = \frac{2}{\sqrt{5}} \quad \text{and} \quad \sin\phi = \frac{1}{\sqrt{5}},$$

and equations (1) and (2) become

$$m_B g\cos 60^\circ - N\frac{2}{\sqrt{5}} = 0 \tag{3}$$

$$m_B g\sin 60^\circ + N\frac{1}{\sqrt{5}} - mg = 0. \tag{4}$$

Solve this system for m_B and N, using $m = 4$ kg and $g = 9.81$ m/s^2.

$$m_B \approx 3.58 \text{ kg}$$

$$N \approx 19.7 \text{ N}$$