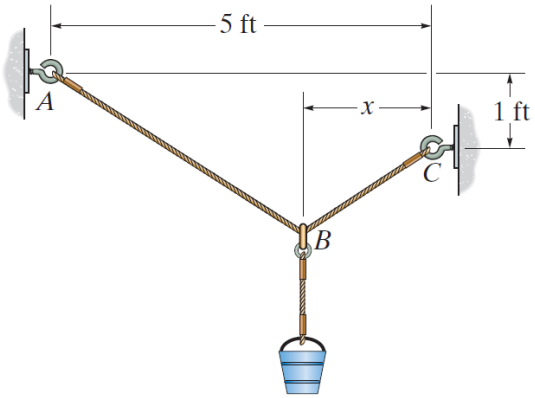
Problem 3-41

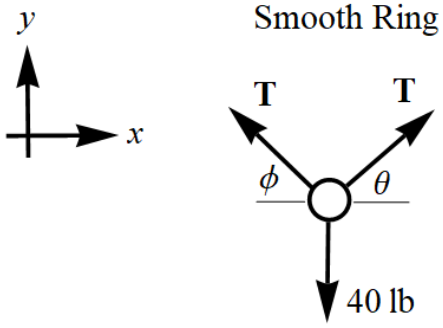
The single elastic cord ABC is used to support the 40-lb load. Determine the position x and the tension in the cord that is required for equilibrium. The cord passes through the smooth ring at B and has an unstretched length of 6 ft and stiffness of $k = 50$ lb/ft.



Prob. 3–41

Solution

Draw a free-body diagram for the smooth ring. Because the ring is smooth, the tension is the same everywhere in the cable.

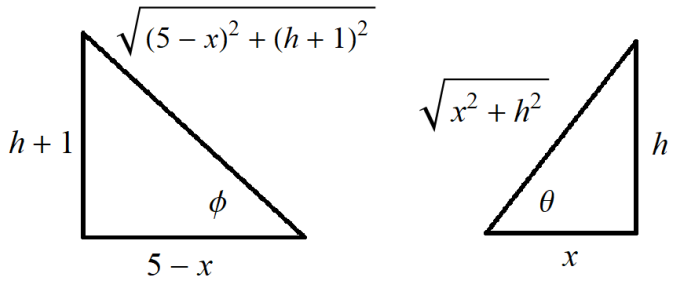


In order for the system to be in equilibrium, the sum of the forces in each direction must be zero.

$$\sum F_x = 0: \qquad T\cos\theta - T\cos\phi = 0 \tag{1}$$

$$\sum F_y = 0: \qquad T\sin\theta + T\sin\phi - 40 = 0 \tag{2}$$

Equation (1) implies that $\theta = \phi$, which means that $\tan\theta = \tan\phi$.



Since $\tan\theta = \tan\phi$,

$$\frac{h}{x} = \frac{h+1}{5-x}. \tag{3}$$

Also, equation (2) becomes

$$2T\sin\theta - 40 = 0 \quad \rightarrow \quad T = \frac{20\text{ lb}}{\sin\theta} = 20\frac{\sqrt{x^2+h^2}}{h}\text{ lb}.$$

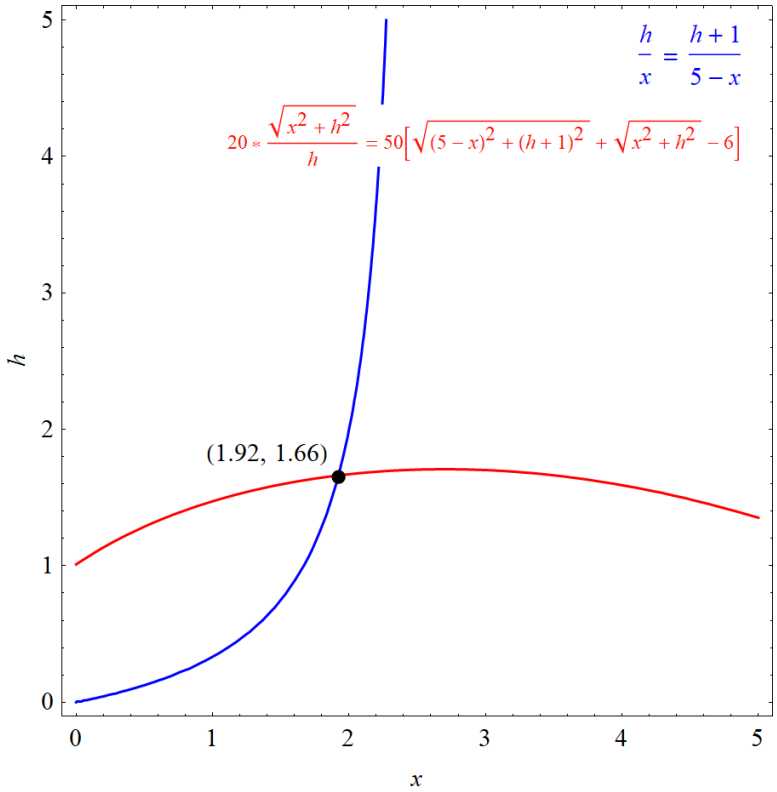
The cable acts as a spring; its tension is equal to the spring constant times the displacement from equilibrium.

$$T = k\Delta x = 50\left[\sqrt{(5-x)^2+(h+1)^2} + \sqrt{x^2+h^2} - 6\right]\text{ lb} \tag{4}$$

As a result,

$$20\frac{\sqrt{x^2+h^2}}{h} = 50\left[\sqrt{(5-x)^2+(h+1)^2} + \sqrt{x^2+h^2} - 6\right]. \tag{5}$$

Plot equations (3) and (5) and find where they intersect to solve for x and h.



With $x \approx 1.92$ ft and $h \approx 1.66$ ft, the tension is $T \approx 30.6$ lb using equation (4).