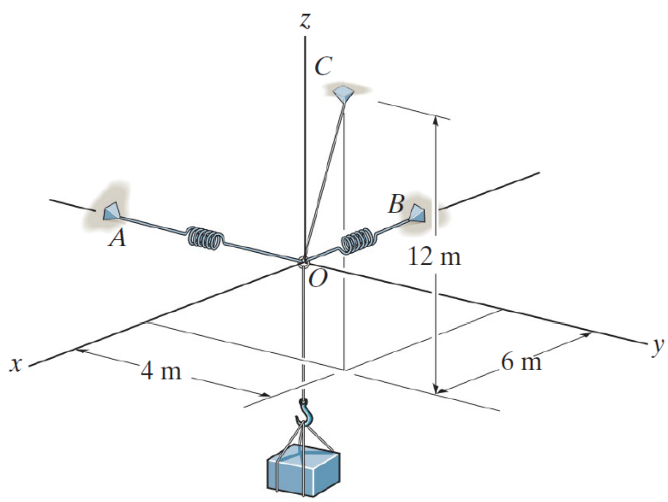
Problem 3-46

Determine the stretch in each of the two springs required to hold the 20-kg crate in the equilibrium position shown. Each spring has an unstretched length of 2 m and a stiffness of $k = 300$ N/m.



Prob. 3–46

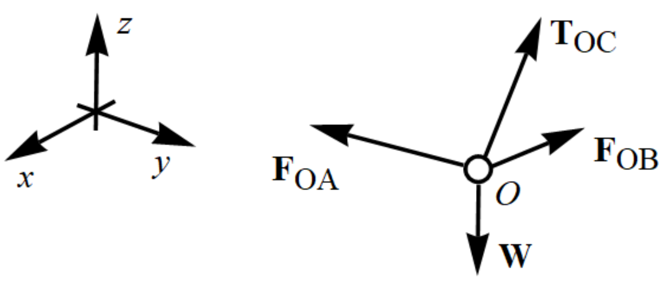
Solution

Write position vectors to point O and point C.

$$\mathbf{r}_O = \langle 0, 0, 0 \rangle \text{ m}$$

$$\mathbf{r}_C = \langle 6, 4, 12 \rangle \text{ m}$$

Draw a free-body diagram for the ring at O.



In order for the system to be in equilibrium, the sum of the forces must be zero.

$$\mathbf{F}_{\text{OA}} + \mathbf{F}_{\text{OB}} + \mathbf{T}_{\text{OC}} + \mathbf{W} = \mathbf{0}$$

$$F_{\text{OA}}\langle 0, -1, 0 \rangle + F_{\text{OB}}\langle -1, 0, 0 \rangle + T_{\text{OC}}\hat{\mathbf{u}}_{\text{OC}} + mg\langle 0, 0, -1 \rangle = \mathbf{0}$$

$$k(x_A - 2)\langle 0, -1, 0 \rangle + k(x_B - 2)\langle -1, 0, 0 \rangle + T_{\text{OC}}\frac{\mathbf{r}_C - \mathbf{r}_O}{|\mathbf{r}_C - \mathbf{r}_O|} + mg\langle 0, 0, -1 \rangle = \mathbf{0}$$

Continue the simplification.

$$k(x_A - 2)\langle 0, -1, 0\rangle + k(x_B - 2)\langle -1, 0, 0\rangle + T_{\text{OC}}\frac{\langle 6-0, 4-0, 12-0\rangle}{\sqrt{(6-0)^2 + (4-0)^2 + (12-0)^2}} + mg\langle 0, 0, -1\rangle = \mathbf{0}$$

$$k(x_A - 2)\langle 0, -1, 0\rangle + k(x_B - 2)\langle -1, 0, 0\rangle + T_{\text{OC}}\left\langle \frac{3}{7}, \frac{2}{7}, \frac{6}{7}\right\rangle + mg\langle 0, 0, -1\rangle = \mathbf{0}$$

$$\left\langle -k(x_B - 2) + \frac{3}{7}T_{\text{OC}},\ -k(x_A - 2) + \frac{2}{7}T_{\text{OC}},\ \frac{6}{7}T_{\text{OC}} - mg\right\rangle = \langle 0, 0, 0\rangle$$

Match the components to get a system of equations.

$$\left.\begin{aligned} -k(x_B - 2) + \frac{3}{7}T_{\text{OC}} &= 0 \\ -k(x_A - 2) + \frac{2}{7}T_{\text{OC}} &= 0 \\ \frac{6}{7}T_{\text{OC}} - mg &= 0 \end{aligned}\right\}$$

Solving it yields

$$T_{\text{OC}} = \frac{7mg}{6}$$

$$x_A = \frac{mg}{3k} + 2$$

$$x_B = \frac{mg}{2k} + 2.$$

Therefore, since $m = 20$ kg and $g = 9.81$ m/s^2 and $k = 300$ N/m, the spring displacements are

$$x_A - 2 \approx 0.218 \text{ m}$$

$$x_B - 2 \approx 0.327 \text{ m}.$$