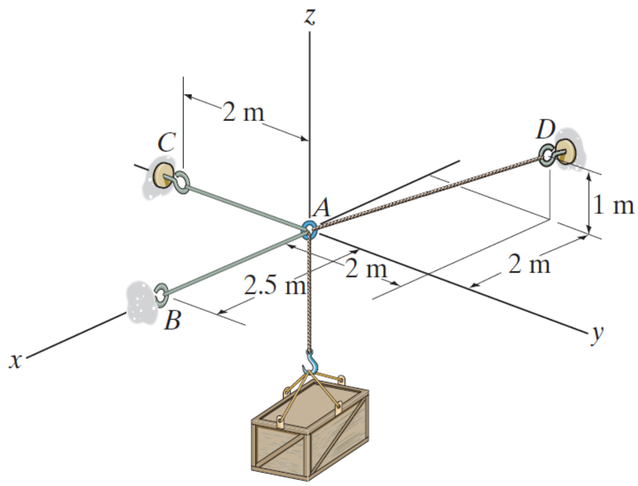
Problem 3-48

Determine the tension in the cables in order to support the 100-kg crate in the equilibrium position shown.



Probs. 3–48/49

Solution

Write position vectors to points A, B, C, and D.

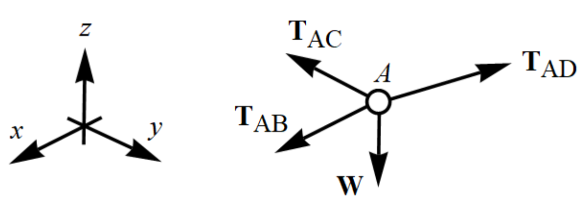
$$\mathbf{r}_A = \langle 0, 0, 0 \rangle \text{ m}$$

$$\mathbf{r}_B = \langle 2.5, 0, 0 \rangle \text{ m}$$

$$\mathbf{r}_C = \langle 0, -2, 0 \rangle \text{ m}$$

$$\mathbf{r}_D = \langle -2, 2, 1 \rangle \text{ m}$$

Draw a free-body diagram for the ring at A.



In order for the system to be in equilibrium, the sum of the forces must be zero.

$$\mathbf{T}_{\text{AB}} + \mathbf{T}_{\text{AC}} + \mathbf{T}_{\text{AD}} + \mathbf{W} = \mathbf{0}$$

$$T_{\text{AB}}\hat{\mathbf{u}}_{\text{AB}} + T_{\text{AC}}\hat{\mathbf{u}}_{\text{AC}} + T_{\text{AD}}\hat{\mathbf{u}}_{\text{AD}} + mg\langle 0, 0, -1 \rangle = \mathbf{0}$$

$$T_{\text{AB}}\frac{\mathbf{r}_B - \mathbf{r}_A}{|\mathbf{r}_B - \mathbf{r}_A|} + T_{\text{AC}}\frac{\mathbf{r}_C - \mathbf{r}_A}{|\mathbf{r}_C - \mathbf{r}_A|} + T_{\text{AD}}\frac{\mathbf{r}_D - \mathbf{r}_A}{|\mathbf{r}_D - \mathbf{r}_A|} + mg\langle 0, 0, -1 \rangle = \mathbf{0}$$

Continue the simplification.

$$T_{\text{AB}}\frac{\langle 2.5-0, 0-0, 0-0\rangle}{\sqrt{(2.5-0)^2+(0-0)^2+(0-0)^2}}+T_{\text{AC}}\frac{\langle 0-0, -2-0, 0-0\rangle}{\sqrt{(0-0)^2+(-2-0)^2+(0-0)^2}} + T_{\text{AD}}\frac{\langle -2-0, 2-0, 1-0\rangle}{\sqrt{(-2-0)^2+(2-0)^2+(1-0)^2}}+mg\langle 0,0,-1\rangle=\mathbf{0}$$

$$T_{\text{AB}}\langle 1,0,0\rangle+T_{\text{AC}}\langle 0,-1,0\rangle+T_{\text{AD}}\left\langle -\frac{2}{3},\frac{2}{3},\frac{1}{3}\right\rangle+mg\langle 0,0,-1\rangle=\mathbf{0}$$

$$\left\langle T_{\text{AB}}-\frac{2}{3}T_{\text{AD}},\ -T_{\text{AC}}+\frac{2}{3}T_{\text{AD}},\ \frac{1}{3}T_{\text{AD}}-mg\right\rangle=\langle 0,0,0\rangle$$

Match the components to get a system of equations.

$$\left.\begin{aligned} T_{\text{AB}}-\frac{2}{3}T_{\text{AD}}&=0\\ -T_{\text{AC}}+\frac{2}{3}T_{\text{AD}}&=0\\ \frac{1}{3}T_{\text{AD}}-mg&=0 \end{aligned}\right\}$$

Solving it yields

$$T_{\text{AB}}=2mg$$

$$T_{\text{AC}}=2mg$$

$$T_{\text{AD}}=3mg.$$

Therefore, since $m=100$ kg and $g=9.81$ m/s^2, the tensions are

$$T_{\text{AB}}\approx 1.96\times 10^3\text{ N}$$

$$T_{\text{AC}}\approx 1.96\times 10^3\text{ N}$$

$$T_{\text{AD}}\approx 2.94\times 10^3\text{ N}.$$