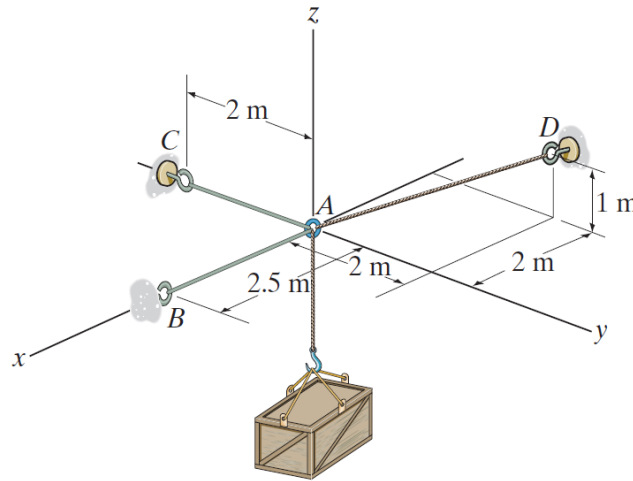


Problem 3-49

Determine the maximum mass of the crate so that the tension developed in any cable does not exceed 3 kN.



Probs. 3-48/49

[TYPO: This should be “exceed.”]

Solution

Write position vectors to points A , B , C , and D .

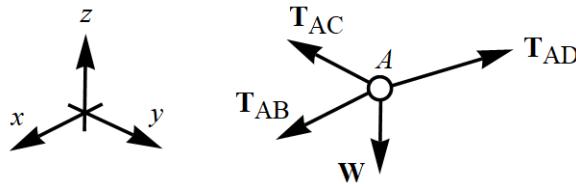
$$\mathbf{r}_A = \langle 0, 0, 0 \rangle \text{ m}$$

$$\mathbf{r}_B = \langle 2.5, 0, 0 \rangle \text{ m}$$

$$\mathbf{r}_C = \langle 0, -2, 0 \rangle \text{ m}$$

$$\mathbf{r}_D = \langle -2, 2, 1 \rangle \text{ m}$$

Draw a free-body diagram for the ring at A .



In order for the system to be in equilibrium, the sum of the forces must be zero.

$$\mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + \mathbf{W} = \mathbf{0}$$

$$T_{AB}\hat{\mathbf{u}}_{AB} + T_{AC}\hat{\mathbf{u}}_{AC} + T_{AD}\hat{\mathbf{u}}_{AD} + mg\langle 0, 0, -1 \rangle = \mathbf{0}$$

$$T_{AB} \frac{\mathbf{r}_B - \mathbf{r}_A}{|\mathbf{r}_B - \mathbf{r}_A|} + T_{AC} \frac{\mathbf{r}_C - \mathbf{r}_A}{|\mathbf{r}_C - \mathbf{r}_A|} + T_{AD} \frac{\mathbf{r}_D - \mathbf{r}_A}{|\mathbf{r}_D - \mathbf{r}_A|} + mg\langle 0, 0, -1 \rangle = \mathbf{0}$$

Continue the simplification.

$$T_{AB} \frac{\langle 2.5 - 0, 0 - 0, 0 - 0 \rangle}{\sqrt{(2.5 - 0)^2 + (0 - 0)^2 + (0 - 0)^2}} + T_{AC} \frac{\langle 0 - 0, -2 - 0, 0 - 0 \rangle}{\sqrt{(0 - 0)^2 + (-2 - 0)^2 + (0 - 0)^2}} \\ + T_{AD} \frac{\langle -2 - 0, 2 - 0, 1 - 0 \rangle}{\sqrt{(-2 - 0)^2 + (2 - 0)^2 + (1 - 0)^2}} + mg\langle 0, 0, -1 \rangle = \mathbf{0}$$

$$T_{AB}\langle 1, 0, 0 \rangle + T_{AC}\langle 0, -1, 0 \rangle + T_{AD} \left\langle -\frac{2}{3}, \frac{2}{3}, \frac{1}{3} \right\rangle + mg\langle 0, 0, -1 \rangle = \mathbf{0}$$

$$\left\langle T_{AB} - \frac{2}{3}T_{AD}, -T_{AC} + \frac{2}{3}T_{AD}, \frac{1}{3}T_{AD} - mg \right\rangle = \langle 0, 0, 0 \rangle$$

Match the components to get a system of equations.

$$\left. \begin{aligned} T_{AB} - \frac{2}{3}T_{AD} &= 0 \\ -T_{AC} + \frac{2}{3}T_{AD} &= 0 \\ \frac{1}{3}T_{AD} - mg &= 0 \end{aligned} \right\}$$

In the previous problem it was found that the highest tension occurs in cable AD . Set $T_{AD} = 3000$ N and solve the system for T_{AB} , T_{AC} , and m .

$$T_{AB} = 2.00 \times 10^3 \text{ N}$$

$$T_{AC} = 2.00 \times 10^3 \text{ N}$$

$$m = \frac{1000}{g} \text{ N}$$

Therefore, since $g = 9.81$ m/s², the maximum mass is

$$m \approx 102 \text{ kg.}$$