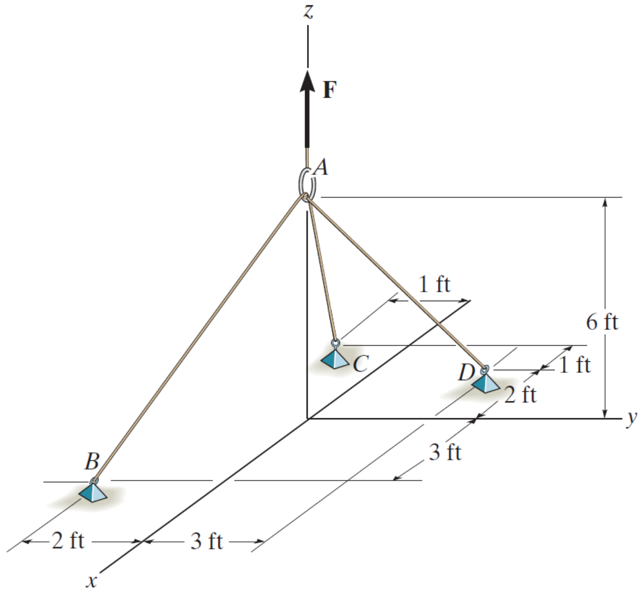
Problem 3-50

Determine the force in each cable if $F = 500$ lb.



Probs. 3–50/51

Solution

Write position vectors to points A, B, C, and D.

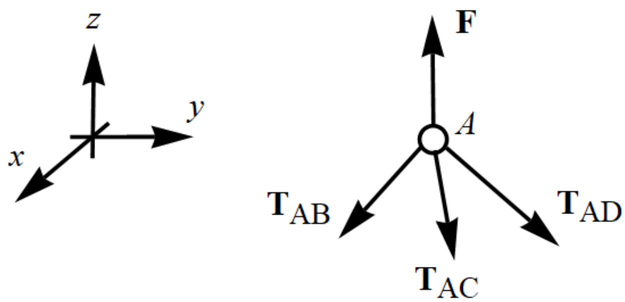
$$\mathbf{r}_A = \langle 0, 0, 6 \rangle \text{ ft}$$

$$\mathbf{r}_B = \langle 3, -2, 0 \rangle \text{ ft}$$

$$\mathbf{r}_C = \langle -3, -1, 0 \rangle \text{ ft}$$

$$\mathbf{r}_D = \langle -2, 3, 0 \rangle \text{ ft}$$

Draw a free-body diagram for the ring at A.



In order for the system to be in equilibrium, the sum of the forces must be zero.

$$\mathbf{T}_{\text{AB}} + \mathbf{T}_{\text{AC}} + \mathbf{T}_{\text{AD}} + \mathbf{F} = \mathbf{0}$$

$$T_{\text{AB}}\hat{\mathbf{u}}_{\text{AB}} + T_{\text{AC}}\hat{\mathbf{u}}_{\text{AC}} + T_{\text{AD}}\hat{\mathbf{u}}_{\text{AD}} + F\langle 0,0,1\rangle = \mathbf{0}$$

$$T_{\text{AB}}\frac{\mathbf{r}_B - \mathbf{r}_A}{|\mathbf{r}_B - \mathbf{r}_A|} + T_{\text{AC}}\frac{\mathbf{r}_C - \mathbf{r}_A}{|\mathbf{r}_C - \mathbf{r}_A|} + T_{\text{AD}}\frac{\mathbf{r}_D - \mathbf{r}_A}{|\mathbf{r}_D - \mathbf{r}_A|} + F\langle 0,0,1\rangle = \mathbf{0}$$

$$T_{\text{AB}}\frac{\langle 3-0, -2-0, 0-6\rangle}{\sqrt{(3-0)^2+(-2-0)^2+(0-6)^2}} + T_{\text{AC}}\frac{\langle -3-0, -1-0, 0-6\rangle}{\sqrt{(-3-0)^2+(-1-0)^2+(0-6)^2}}$$
$$+ T_{\text{AD}}\frac{\langle -2-0, 3-0, 0-6\rangle}{\sqrt{(-2-0)^2+(3-0)^2+(0-6)^2}} + F\langle 0,0,1\rangle = \mathbf{0}$$

$$T_{\text{AB}}\left\langle \frac{3}{7}, -\frac{2}{7}, -\frac{6}{7}\right\rangle + T_{\text{AC}}\left\langle -\frac{3}{\sqrt{46}}, -\frac{1}{\sqrt{46}}, -3\sqrt{\frac{2}{23}}\right\rangle + T_{\text{AD}}\left\langle -\frac{2}{7}, \frac{3}{7}, -\frac{6}{7}\right\rangle + F\langle 0,0,1\rangle = \mathbf{0}$$

$$\left\langle \frac{3}{7}T_{\text{AB}} - \frac{3}{\sqrt{46}}T_{\text{AC}} - \frac{2}{7}T_{\text{AD}},\ -\frac{2}{7}T_{\text{AB}} - \frac{1}{\sqrt{46}}T_{\text{AC}} + \frac{3}{7}T_{\text{AD}},\ -\frac{6}{7}T_{\text{AB}} - 3\sqrt{\frac{2}{23}}T_{\text{AC}} - \frac{6}{7}T_{\text{AD}} + F\right\rangle = \langle 0,0,0\rangle$$

Match the components to get a system of equations.

$$\left.\begin{aligned} \frac{3}{7}T_{\text{AB}} - \frac{3}{\sqrt{46}}T_{\text{AC}} - \frac{2}{7}T_{\text{AD}} &= 0 \\ -\frac{2}{7}T_{\text{AB}} - \frac{1}{\sqrt{46}}T_{\text{AC}} + \frac{3}{7}T_{\text{AD}} &= 0 \\ -\frac{6}{7}T_{\text{AB}} - 3\sqrt{\frac{2}{23}}T_{\text{AC}} - \frac{6}{7}T_{\text{AD}} + F &= 0 \end{aligned}\right\}$$

Solving it yields

$$T_{\text{AB}} = \frac{77}{150}F$$

$$T_{\text{AC}} = \frac{1}{15}\sqrt{\frac{23}{2}}F$$

$$T_{\text{AD}} = \frac{21}{50}F.$$

Therefore, since $F = 500$ lb,

$$T_{\text{AB}} = \frac{770}{3}\text{ lb} \approx 257\text{ lb}$$

$$T_{\text{AC}} = \frac{50\sqrt{46}}{3}\text{ lb} \approx 113\text{ lb}$$

$$T_{\text{AD}} = 210\text{ lb}.$$