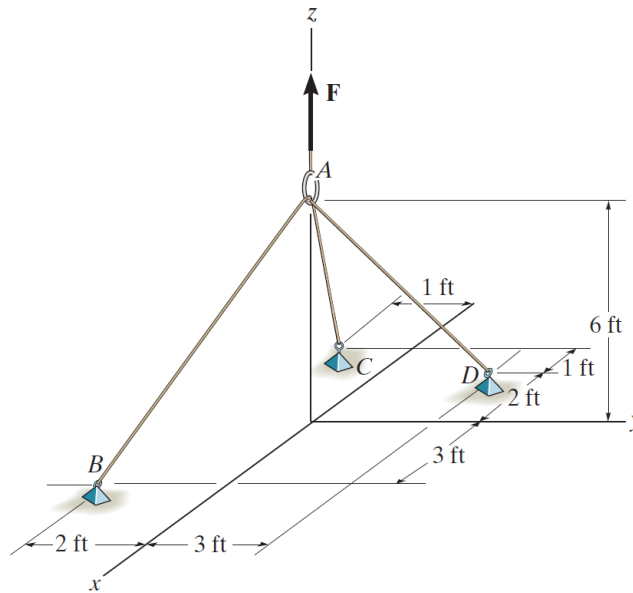


Problem 3-51

Determine the greatest force \mathbf{F} that can be applied to the ring if each cable can support a maximum force of 800 lb.



Probs. 3-50/51

Solution

Write position vectors to points A , B , C , and D .

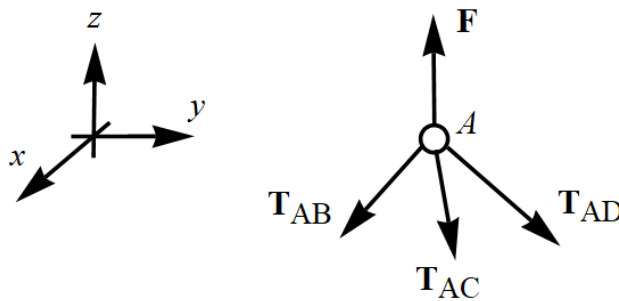
$$\mathbf{r}_A = \langle 0, 0, 6 \rangle \text{ ft}$$

$$\mathbf{r}_B = \langle 3, -2, 0 \rangle \text{ ft}$$

$$\mathbf{r}_C = \langle -3, -1, 0 \rangle \text{ ft}$$

$$\mathbf{r}_D = \langle -2, 3, 0 \rangle \text{ ft}$$

Draw a free-body diagram for the ring at A .



In order for the system to be in equilibrium, the sum of the forces must be zero.

$$\mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + \mathbf{F} = \mathbf{0}$$

$$T_{AB}\hat{\mathbf{u}}_{AB} + T_{AC}\hat{\mathbf{u}}_{AC} + T_{AD}\hat{\mathbf{u}}_{AD} + F\langle 0, 0, 1 \rangle = \mathbf{0}$$

$$T_{AB}\frac{\mathbf{r}_B - \mathbf{r}_A}{|\mathbf{r}_B - \mathbf{r}_A|} + T_{AC}\frac{\mathbf{r}_C - \mathbf{r}_A}{|\mathbf{r}_C - \mathbf{r}_A|} + T_{AD}\frac{\mathbf{r}_D - \mathbf{r}_A}{|\mathbf{r}_D - \mathbf{r}_A|} + F\langle 0, 0, 1 \rangle = \mathbf{0}$$

$$T_{AB}\frac{\langle 3-0, -2-0, 0-6 \rangle}{\sqrt{(3-0)^2 + (-2-0)^2 + (0-6)^2}} + T_{AC}\frac{\langle -3-0, -1-0, 0-6 \rangle}{\sqrt{(-3-0)^2 + (-1-0)^2 + (0-6)^2}} \\ + T_{AD}\frac{\langle -2-0, 3-0, 0-6 \rangle}{\sqrt{(-2-0)^2 + (3-0)^2 + (0-6)^2}} + F\langle 0, 0, 1 \rangle = \mathbf{0}$$

$$T_{AB}\left\langle \frac{3}{7}, -\frac{2}{7}, -\frac{6}{7} \right\rangle + T_{AC}\left\langle -\frac{3}{\sqrt{46}}, -\frac{1}{\sqrt{46}}, -3\sqrt{\frac{2}{23}} \right\rangle + T_{AD}\left\langle -\frac{2}{7}, \frac{3}{7}, -\frac{6}{7} \right\rangle + F\langle 0, 0, 1 \rangle = \mathbf{0}$$

$$\left\langle \frac{3}{7}T_{AB} - \frac{3}{\sqrt{46}}T_{AC} - \frac{2}{7}T_{AD}, -\frac{2}{7}T_{AB} - \frac{1}{\sqrt{46}}T_{AC} + \frac{3}{7}T_{AD}, -\frac{6}{7}T_{AB} - 3\sqrt{\frac{2}{23}}T_{AC} - \frac{6}{7}T_{AD} + F \right\rangle = \langle 0, 0, 0 \rangle$$

Match the components to get a system of equations.

$$\left. \begin{aligned} \frac{3}{7}T_{AB} - \frac{3}{\sqrt{46}}T_{AC} - \frac{2}{7}T_{AD} &= 0 \\ -\frac{2}{7}T_{AB} - \frac{1}{\sqrt{46}}T_{AC} + \frac{3}{7}T_{AD} &= 0 \\ -\frac{6}{7}T_{AB} - 3\sqrt{\frac{2}{23}}T_{AC} - \frac{6}{7}T_{AD} + F &= 0 \end{aligned} \right\}$$

In the previous problem it was found that cable AB has the highest tension. Set $T_{AB} = 800$ lb, then, and solve the system for T_{AC} , T_{AD} , and F .

$$T_{AC} = \frac{4000\sqrt{46}}{77} \text{ lb} \approx 352 \text{ lb}$$

$$T_{AD} = \frac{7200}{11} \text{ lb} \approx 655 \text{ lb}$$

$$F = \frac{120\,000}{77} \text{ lb} \approx 1.56 \times 10^3 \text{ lb}$$