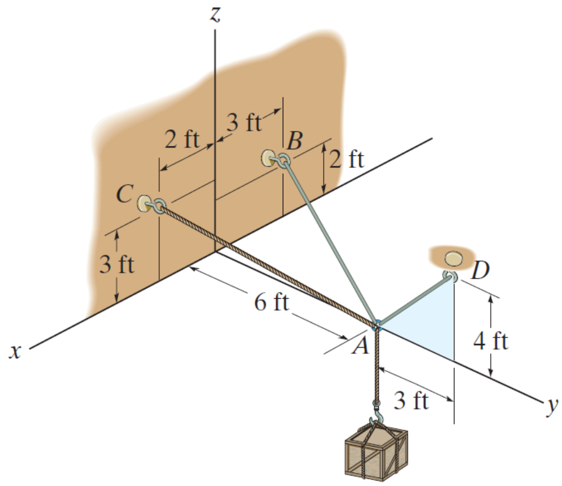
Problem 3-54

Determine the tension developed in each cable for equilibrium of the 300-lb crate.



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Solution

Write position vectors to points A, B, C, and D.

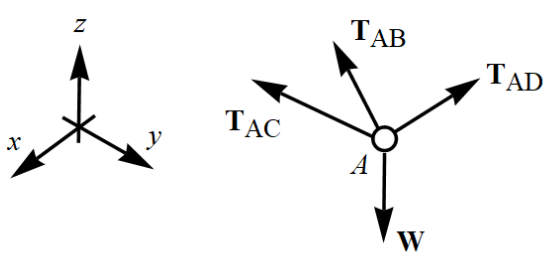
$$\mathbf{r}_A = \langle 0, 6, 0 \rangle \text{ ft}$$

$$\mathbf{r}_B = \langle -3, 0, 2 \rangle \text{ ft}$$

$$\mathbf{r}_C = \langle 2, 0, 3 \rangle \text{ ft}$$

$$\mathbf{r}_D = \langle 0, 9, 4 \rangle \text{ ft}$$

Draw a free-body diagram for the ring at A.



In order for the system to be in equilibrium, the sum of the forces must be zero.

$$\mathbf{T}_{\text{AB}} + \mathbf{T}_{\text{AC}} + \mathbf{T}_{\text{AD}} + \mathbf{W} = \mathbf{0}$$

$$T_{\text{AB}}\hat{\mathbf{u}}_{\text{AB}} + T_{\text{AC}}\hat{\mathbf{u}}_{\text{AC}} + T_{\text{AD}}\hat{\mathbf{u}}_{\text{AD}} + W\langle 0, 0, -1 \rangle = \mathbf{0}$$

$$T_{\text{AB}}\frac{\mathbf{r}_B - \mathbf{r}_A}{|\mathbf{r}_B - \mathbf{r}_A|} + T_{\text{AC}}\frac{\mathbf{r}_C - \mathbf{r}_A}{|\mathbf{r}_C - \mathbf{r}_A|} + T_{\text{AD}}\frac{\mathbf{r}_D - \mathbf{r}_A}{|\mathbf{r}_D - \mathbf{r}_A|} + W\langle 0, 0, -1 \rangle = \mathbf{0}$$

Continue the simplification.

$$T_{\text{AB}}\frac{\langle -3-0, 0-6, 2-0\rangle}{\sqrt{(-3-0)^2+(0-6)^2+(2-0)^2}} + T_{\text{AC}}\frac{\langle 2-0, 0-6, 3-0\rangle}{\sqrt{(2-0)^2+(0-6)^2+(3-0)^2}} + T_{\text{AD}}\frac{\langle 0-0, 9-6, 4-0\rangle}{\sqrt{(0-0)^2+(9-6)^2+(4-0)^2}} + W\langle 0,0,-1\rangle = \mathbf{0}$$

$$T_{\text{AB}}\left\langle -\frac{3}{7}, -\frac{6}{7}, \frac{2}{7}\right\rangle + T_{\text{AC}}\left\langle \frac{2}{7}, -\frac{6}{7}, \frac{3}{7}\right\rangle + T_{\text{AD}}\left\langle 0, \frac{3}{5}, \frac{4}{5}\right\rangle + W\langle 0,0,-1\rangle = \mathbf{0}$$

$$\left\langle -\frac{3}{7}T_{\text{AB}} + \frac{2}{7}T_{\text{AC}},\ -\frac{6}{7}T_{\text{AB}} - \frac{6}{7}T_{\text{AC}} + \frac{3}{5}T_{\text{AD}},\ \frac{2}{7}T_{\text{AB}} + \frac{3}{7}T_{\text{AC}} + \frac{4}{5}T_{\text{AD}} - W\right\rangle = \langle 0,0,0\rangle$$

Match the components to get a system of equations.

$$\left.\begin{aligned} -\frac{3}{7}T_{\text{AB}} + \frac{2}{7}T_{\text{AC}} &= 0 \\ -\frac{6}{7}T_{\text{AB}} - \frac{6}{7}T_{\text{AC}} + \frac{3}{5}T_{\text{AD}} &= 0 \\ \frac{2}{7}T_{\text{AB}} + \frac{3}{7}T_{\text{AC}} + \frac{4}{5}T_{\text{AD}} - W &= 0 \end{aligned}\right\}$$

Solving it yields

$$T_{\text{AB}} = \frac{14W}{53}$$

$$T_{\text{AC}} = \frac{21W}{53}$$

$$T_{\text{AD}} = \frac{50W}{53}.$$

Therefore, since $W = 300$ lb,

$$T_{\text{AB}} = \frac{4200}{53}\text{ lb} \approx 79.2\text{ lb}$$

$$T_{\text{AC}} = \frac{6300}{53}\text{ lb} \approx 119\text{ lb}$$

$$T_{\text{AD}} = \frac{15\,000}{53}\text{ lb} \approx 283\text{ lb}.$$