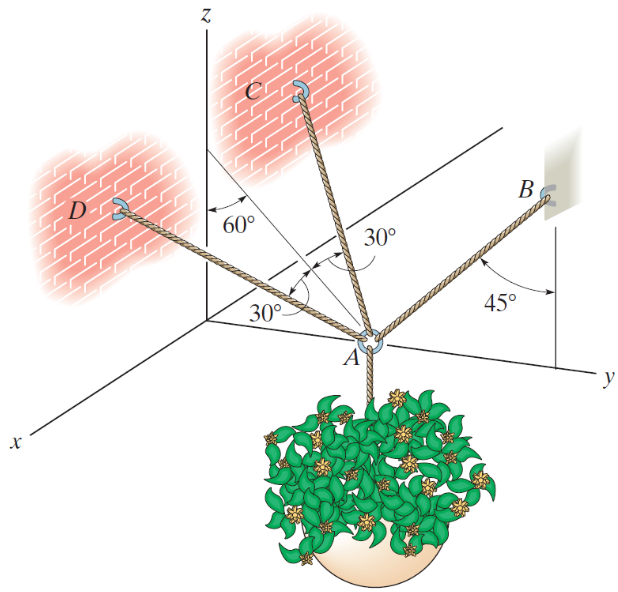
Problem 3-56

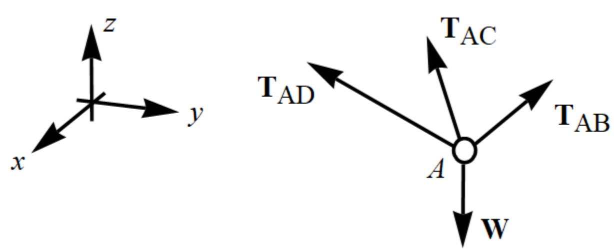
The 25-kg flowerpot is supported at A by the three cords. Determine the force acting in each cord for equilibrium.



Probs. 3–56/57

Solution

Draw a free-body diagram for the ring at A.



In order for the system to be in equilibrium, the sum of the forces must be zero. Note that the angle that cord AB makes with the y-axis is 45°, and the missing angle of the triangle against the brick wall is 30°.

$$\mathbf{T}_{\text{AB}} + \mathbf{T}_{\text{AC}} + \mathbf{T}_{\text{AD}} + \mathbf{W} = \mathbf{0}$$

$$T_{\text{AB}}\hat{\mathbf{u}}_{\text{AB}} + T_{\text{AC}}\hat{\mathbf{u}}_{\text{AC}} + T_{\text{AD}}\hat{\mathbf{u}}_{\text{AD}} + W\langle 0, 0, -1\rangle = \mathbf{0}$$

$$T_{\text{AB}}\langle 0, \cos 45^\circ, \sin 45^\circ\rangle + T_{\text{AC}}\langle -\sin 30^\circ, -\cos 30^\circ \cos 30^\circ, \cos 30^\circ \sin 30^\circ\rangle$$
$$+ T_{\text{AD}}\langle \sin 30^\circ, -\cos 30^\circ \cos 30^\circ, \cos 30^\circ \sin 30^\circ\rangle + mg\langle 0, 0, -1\rangle = \mathbf{0}$$

Combine the vectors on the left side.

$$\langle -T_{\text{AC}} \sin 30^\circ + T_{\text{AD}} \sin 30^\circ,\quad T_{\text{AB}} \cos 45^\circ - T_{\text{AC}} \cos 30^\circ \cos 30^\circ - T_{\text{AD}} \cos 30^\circ \cos 30^\circ,\quad T_{\text{AB}} \sin 45^\circ + T_{\text{AC}} \cos 30^\circ \sin 30^\circ + T_{\text{AD}} \cos 30^\circ \sin 30^\circ - mg\rangle = \langle 0, 0, 0\rangle$$

Match the components to get a system of equations.

$$\left.\begin{aligned} -T_{\text{AC}} \sin 30^\circ + T_{\text{AD}} \sin 30^\circ = 0 \\ T_{\text{AB}} \cos 45^\circ - T_{\text{AC}} \cos 30^\circ \cos 30^\circ - T_{\text{AD}} \cos 30^\circ \cos 30^\circ = 0 \\ T_{\text{AB}} \sin 45^\circ + T_{\text{AC}} \cos 30^\circ \sin 30^\circ + T_{\text{AD}} \cos 30^\circ \sin 30^\circ - mg = 0 \end{aligned}\right\}$$

Solving the system yields

$$T_{\text{AB}} = \frac{3\sqrt{2}}{3+\sqrt{3}} mg$$

$$T_{\text{AC}} = \frac{2}{3+\sqrt{3}} mg$$

$$T_{\text{AD}} = \frac{2}{3+\sqrt{3}} mg.$$

Therefore, since $m = 25$ kg and $g = 9.81$ m/s^2,

$$T_{\text{AB}} \approx 220.\ \text{N}$$

$$T_{\text{AC}} \approx 104\ \text{N}$$

$$T_{\text{AD}} \approx 104\ \text{N}.$$