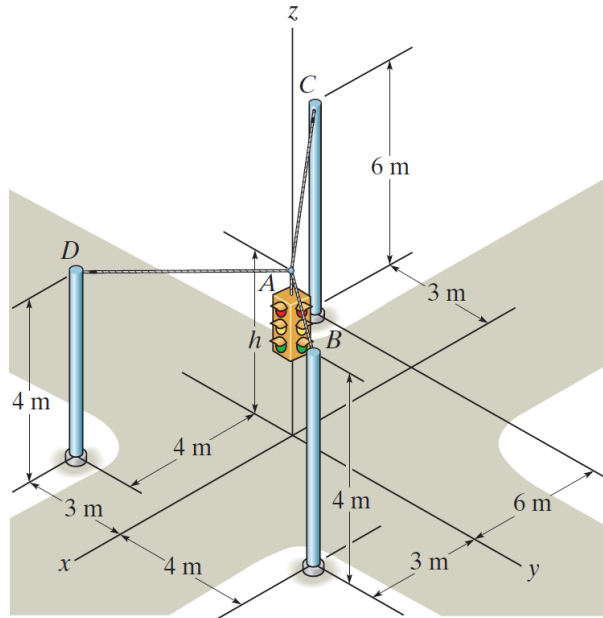


Problem 3-58

Determine the tension developed in the three cables required to support the traffic light, which has a mass of 15 kg. Take $h = 4$ m.



Prob. 3-58

Solution

Write position vectors to points A , B , C , and D .

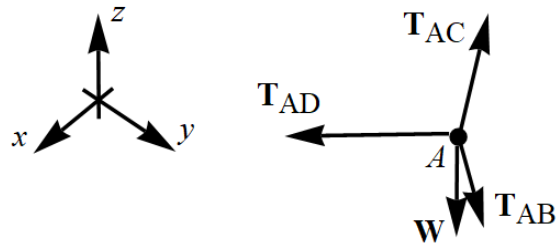
$$\mathbf{r}_A = \langle 0, 0, h \rangle$$

$$\mathbf{r}_B = \langle 3, 4, 4 \rangle \text{ m}$$

$$\mathbf{r}_C = \langle -6, -3, 6 \rangle \text{ m}$$

$$\mathbf{r}_D = \langle 4, -3, 4 \rangle \text{ m}$$

Draw a free-body diagram for the knot at A .



In order for the system to be in equilibrium, the sum of the forces must be zero.

$$\mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + \mathbf{W} = \mathbf{0}$$

$$T_{AB}\hat{\mathbf{u}}_{AB} + T_{AC}\hat{\mathbf{u}}_{AC} + T_{AD}\hat{\mathbf{u}}_{AD} + W\langle 0, 0, -1 \rangle = \mathbf{0}$$

$$T_{AB}\frac{\mathbf{r}_B - \mathbf{r}_A}{|\mathbf{r}_B - \mathbf{r}_A|} + T_{AC}\frac{\mathbf{r}_C - \mathbf{r}_A}{|\mathbf{r}_C - \mathbf{r}_A|} + T_{AD}\frac{\mathbf{r}_D - \mathbf{r}_A}{|\mathbf{r}_D - \mathbf{r}_A|} + mg\langle 0, 0, -1 \rangle = \mathbf{0}$$

$$T_{AB}\frac{\langle 3-0, 4-0, 4-h \rangle}{\sqrt{(3-0)^2 + (4-0)^2 + (4-h)^2}} + T_{AC}\frac{\langle -6-0, -3-0, 6-h \rangle}{\sqrt{(-6-0)^2 + (-3-0)^2 + (6-h)^2}} \\ + T_{AD}\frac{\langle 4-0, -3-0, 4-h \rangle}{\sqrt{(4-0)^2 + (-3-0)^2 + (4-h)^2}} + mg\langle 0, 0, -1 \rangle = \mathbf{0}$$

$$\frac{T_{AB}}{\sqrt{25 + (4-h)^2}}\langle 3, 4, 4-h \rangle + \frac{T_{AC}}{\sqrt{45 + (6-h)^2}}\langle -6, -3, 6-h \rangle \\ + \frac{T_{AD}}{\sqrt{25 + (4-h)^2}}\langle 4, -3, 4-h \rangle + mg\langle 0, 0, -1 \rangle = \mathbf{0}$$

$$\left\langle \frac{3}{\sqrt{25 + (4-h)^2}}T_{AB} - \frac{6}{\sqrt{45 + (6-h)^2}}T_{AC} + \frac{4}{\sqrt{25 + (4-h)^2}}T_{AD}, \right. \\ \left. \frac{4}{\sqrt{25 + (4-h)^2}}T_{AB} - \frac{3}{\sqrt{45 + (6-h)^2}}T_{AC} - \frac{3}{\sqrt{25 + (4-h)^2}}T_{AD} \right. \\ \left. \frac{4-h}{\sqrt{25 + (4-h)^2}}T_{AB} + \frac{6-h}{\sqrt{45 + (6-h)^2}}T_{AC} + \frac{4-h}{\sqrt{25 + (4-h)^2}}T_{AD} - mg \right\rangle = \langle 0, 0, 0 \rangle$$

Match the components to get a system of equations.

$$\left. \begin{aligned} \frac{3}{\sqrt{25 + (4-h)^2}}T_{AB} - \frac{6}{\sqrt{45 + (6-h)^2}}T_{AC} + \frac{4}{\sqrt{25 + (4-h)^2}}T_{AD} &= 0 \\ \frac{4}{\sqrt{25 + (4-h)^2}}T_{AB} - \frac{3}{\sqrt{45 + (6-h)^2}}T_{AC} - \frac{3}{\sqrt{25 + (4-h)^2}}T_{AD} &= 0 \\ \frac{4-h}{\sqrt{25 + (4-h)^2}}T_{AB} + \frac{6-h}{\sqrt{45 + (6-h)^2}}T_{AC} + \frac{4-h}{\sqrt{25 + (4-h)^2}}T_{AD} - mg &= 0 \end{aligned} \right\}$$

Solving it and plugging in $m = 15 \text{ kg}$ and $g = 9.81 \text{ m/s}^2$ and $h = 4 \text{ m}$ yields

$$T_{AB} = \frac{3\sqrt{41 + h(h-8)}}{33 - 7h}mg \approx 441 \text{ N}$$

$$T_{AC} = \frac{5\sqrt{81 + h(h-12)}}{2(33 - 7h)}mg \approx 515 \text{ N}$$

$$T_{AD} = \frac{3\sqrt{41 + h(h-8)}}{2(33 - 7h)}mg \approx 221 \text{ N.}$$