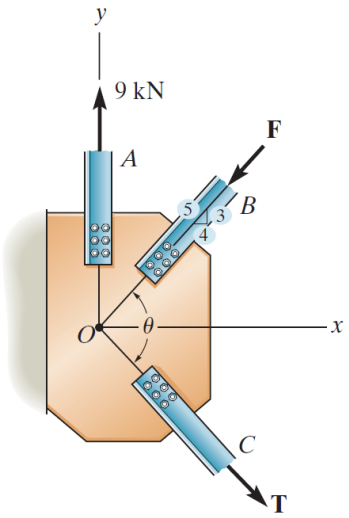
Problem 3-6

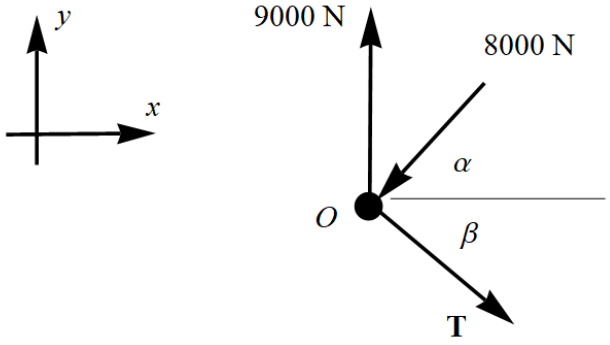
The gusset plate is subjected to the forces of three members. Determine the tension force in member C and its angle θ for equilibrium. The forces are concurrent at point O. Take $F = 8$ kN.



Probs. 3–5/6

Solution

Draw a free-body diagram for the plate.



Begin by finding α, the angle that **F** makes with the x-axis.

$$\tan\alpha = \frac{3}{4} \quad \rightarrow \quad \alpha = \tan^{-1}\left(\frac{3}{4}\right) \approx 36.9^\circ$$

The angle that **T** makes with the x-axis is then

$$\beta = \theta - \alpha.$$

In order for the plate to be in equilibrium, the sum of the forces in each direction must be zero.

$$\sum F_x = 0: \qquad T\cos\beta - 8000\cos\alpha = 0$$

$$\sum F_y = 0: \qquad 9000 - 8000\sin\alpha - T\sin\beta = 0$$

Solve for the terms with T.

$$T\cos\beta = 8000\cos\alpha \tag{1}$$

$$T\sin\beta = 9000 - 8000\sin\alpha \tag{2}$$

Square both sides of each equation and then add them respectively to eliminate β.

$$\begin{aligned} T^2(\cos^2\beta + \sin^2\beta) &= (8000\cos\alpha)^2 + (9000 - 8000\sin\alpha)^2 \\ T^2(1) &= (8000\cos\alpha)^2 + (9000 - 8000\sin\alpha)^2 \\ T &= \sqrt{(8000\cos\alpha)^2 + (9000 - 8000\sin\alpha)^2}\ \text{N} \\ &\approx 7.66\times 10^3\ \text{N} \end{aligned}$$

Substitute this value for T into equation (1) to determine β.

$$\cos\beta = \frac{8000\cos\alpha}{T} \quad \rightarrow \quad \beta \approx 33.3^\circ$$

Therefore, since $\theta = \alpha + \beta$,

$$\theta \approx 70.1^\circ.$$