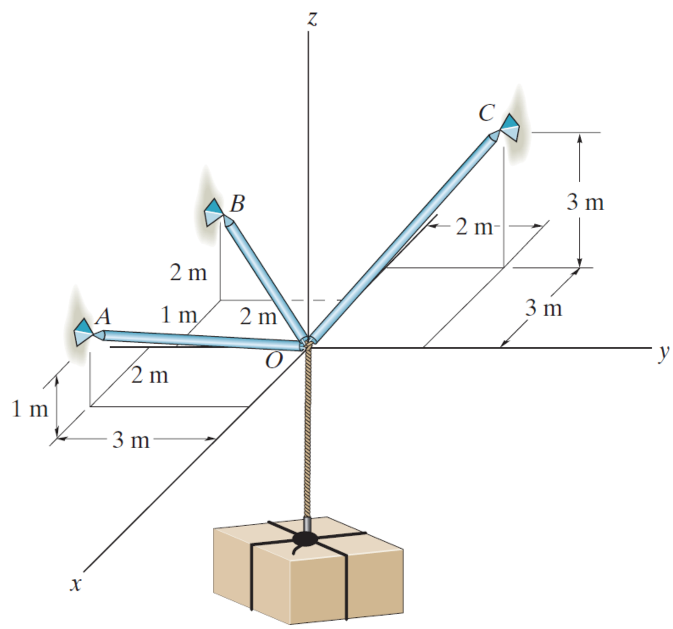
Problem 3-62

If the maximum force in each rod can not exceed 1500 N, determine the greatest mass of the crate that can be supported.



Prob. 3–62

Solution

Write position vectors to points O, A, B, and C.

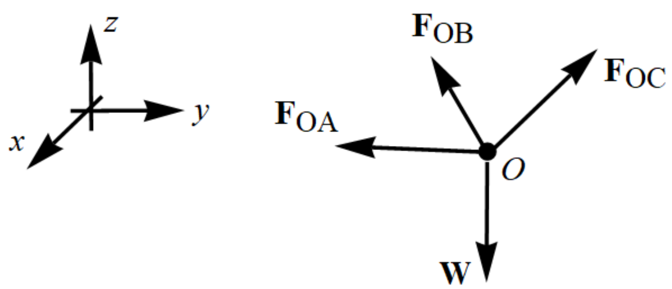
$$\mathbf{r}_O = \langle 0, 0, 0 \rangle \text{ m}$$

$$\mathbf{r}_A = \langle 2, -3, 1 \rangle \text{ m}$$

$$\mathbf{r}_B = \langle -1, -2, 2 \rangle \text{ m}$$

$$\mathbf{r}_C = \langle -3, 2, 3 \rangle \text{ m}$$

Draw a free-body diagram for the knot at O.



In order for the system to be in equilibrium, the sum of the forces must be zero.

$$\mathbf{F}_{\text{OA}} + \mathbf{F}_{\text{OB}} + \mathbf{F}_{\text{OC}} + \mathbf{W} = \mathbf{0}$$

$$F_{\text{OA}}\hat{\mathbf{u}}_{\text{OA}} + F_{\text{OB}}\hat{\mathbf{u}}_{\text{OB}} + F_{\text{OC}}\hat{\mathbf{u}}_{\text{OC}} + W\langle 0, 0, -1\rangle = \mathbf{0}$$

$$F_{\text{OA}}\frac{\mathbf{r}_A - \mathbf{r}_O}{|\mathbf{r}_A - \mathbf{r}_O|} + F_{\text{OB}}\frac{\mathbf{r}_B - \mathbf{r}_O}{|\mathbf{r}_B - \mathbf{r}_O|} + F_{\text{OC}}\frac{\mathbf{r}_C - \mathbf{r}_O}{|\mathbf{r}_C - \mathbf{r}_O|} + mg\langle 0, 0, -1\rangle = \mathbf{0}$$

$$F_{\text{OA}}\frac{\langle 2-0, -3-0, 1-0\rangle}{\sqrt{(2-0)^2 + (-3-0)^2 + (1-0)^2}} + F_{\text{OB}}\frac{\langle -1-0, -2-0, 2-0\rangle}{\sqrt{(-1-0)^2 + (-2-0)^2 + (2-0)^2}} + F_{\text{OC}}\frac{\langle -3-0, 2-0, 3-0\rangle}{\sqrt{(-3-0)^2 + (2-0)^2 + (3-0)^2}} + mg\langle 0, 0, -1\rangle = \mathbf{0}$$

$$F_{\text{OA}}\left\langle \sqrt{\frac{2}{7}}, -\frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}} \right\rangle + F_{\text{OB}}\left\langle -\frac{1}{3}, -\frac{2}{3}, \frac{2}{3} \right\rangle + F_{\text{OC}}\left\langle -\frac{3}{\sqrt{22}}, \sqrt{\frac{2}{11}}, \frac{3}{\sqrt{22}} \right\rangle + mg\langle 0, 0, -1\rangle = \mathbf{0}$$

$$\left\langle \sqrt{\frac{2}{7}}F_{\text{OA}} - \frac{1}{3}F_{\text{OB}} - \frac{3}{\sqrt{22}}F_{\text{OC}},\ -\frac{3}{\sqrt{14}}F_{\text{OA}} - \frac{2}{3}F_{\text{OB}} + \sqrt{\frac{2}{11}}F_{\text{OC}},\ \frac{1}{\sqrt{14}}F_{\text{OA}} + \frac{2}{3}F_{\text{OB}} + \frac{3}{\sqrt{22}}F_{\text{OC}} - mg \right\rangle = \langle 0, 0, 0\rangle$$

Match the components to get a system of equations.

$$\left.\begin{aligned} \sqrt{\frac{2}{7}}F_{\text{OA}} - \frac{1}{3}F_{\text{OB}} - \frac{3}{\sqrt{22}}F_{\text{OC}} &= 0 \\ -\frac{3}{\sqrt{14}}F_{\text{OA}} - \frac{2}{3}F_{\text{OB}} + \sqrt{\frac{2}{11}}F_{\text{OC}} &= 0 \\ \frac{1}{\sqrt{14}}F_{\text{OA}} + \frac{2}{3}F_{\text{OB}} + \frac{3}{\sqrt{22}}F_{\text{OC}} - mg &= 0 \end{aligned}\right\}$$

Solving the system yields

$$F_{\text{OA}} = \frac{8\sqrt{14}}{19}mg \approx 1.58mg$$

$$F_{\text{OB}} = -\frac{15}{19}mg \approx -0.789mg$$

$$F_{\text{OC}} = \frac{7\sqrt{22}}{19}mg \approx 1.73mg.$$

The minus sign in F_{OB} indicates that $\mathbf{F}_{\text{OB}}$ points in the opposite sense than shown in the free-body diagram. F_{OC} is highest, so set $F_{\text{OC}} = 1500$ N and solve the system again for F_{OA}, F_{OB}, and m. Use $g = 9.81$ m/s^2.

$$F_{\text{OA}} = \frac{12\,000}{\sqrt{77}} \text{ N} \approx 1370 \text{ N}$$

$$F_{\text{OB}} = -\frac{11\,250}{7}\sqrt{\frac{2}{11}} \text{ N} \approx -685 \text{ N}$$

$$m = \frac{14\,250}{7g}\sqrt{\frac{2}{11}} \text{ N} \approx 88.5 \text{ kg}$$