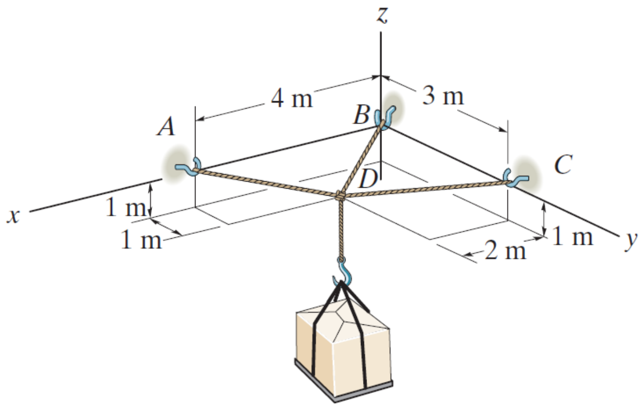
Problem 3-63

The crate has a mass of 130 kg. Determine the tension developed in each cable for equilibrium.



Prob. 3–63

Solution

Write position vectors to points A, B, C, and D.

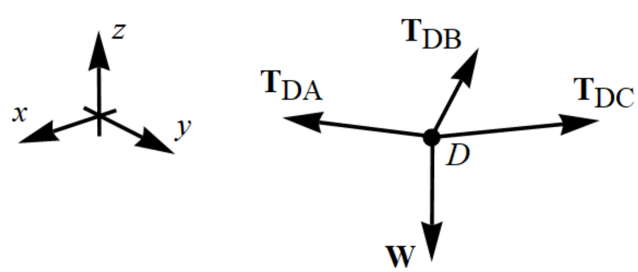
$$\mathbf{r}_A = \langle 4, 0, 0 \rangle \text{ m}$$

$$\mathbf{r}_B = \langle 0, 0, 0 \rangle \text{ m}$$

$$\mathbf{r}_C = \langle 0, 3, 0 \rangle \text{ m}$$

$$\mathbf{r}_D = \langle 2, 1, -1 \rangle \text{ m}$$

Draw a free-body diagram for the knot at D.



In order for the system to be in equilibrium, the sum of the forces must be zero.

$$\mathbf{T}_{\text{DA}} + \mathbf{T}_{\text{DB}} + \mathbf{T}_{\text{DC}} + \mathbf{W} = \mathbf{0}$$

$$T_{\text{DA}}\hat{\mathbf{u}}_{\text{DA}} + T_{\text{DB}}\hat{\mathbf{u}}_{\text{DB}} + T_{\text{DC}}\hat{\mathbf{u}}_{\text{DC}} + W\langle 0, 0, -1 \rangle = \mathbf{0}$$

$$T_{\text{DA}}\frac{\mathbf{r}_A - \mathbf{r}_D}{|\mathbf{r}_A - \mathbf{r}_D|} + T_{\text{DB}}\frac{\mathbf{r}_B - \mathbf{r}_D}{|\mathbf{r}_B - \mathbf{r}_D|} + T_{\text{DC}}\frac{\mathbf{r}_C - \mathbf{r}_D}{|\mathbf{r}_C - \mathbf{r}_D|} + mg\langle 0, 0, -1 \rangle = \mathbf{0}$$

Write out the unit vectors and simplify the left side.

$$T_{\text{DA}}\frac{\langle 4-2, 0-1, 0+1\rangle}{\sqrt{(4-2)^2+(0-1)^2+(0+1)^2}} + T_{\text{DB}}\frac{\langle 0-2, 0-1, 0+1\rangle}{\sqrt{(0-2)^2+(0-1)^2+(0+1)^2}} + T_{\text{DC}}\frac{\langle 0-2, 3-1, 0+1\rangle}{\sqrt{(0-2)^2+(3-1)^2+(0+1)^2}} + mg\langle 0,0,-1\rangle = \mathbf{0}$$

$$T_{\text{DA}}\left\langle \sqrt{\frac{2}{3}}, -\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right\rangle + T_{\text{DB}}\left\langle -\sqrt{\frac{2}{3}}, -\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right\rangle + T_{\text{DC}}\left\langle -\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right\rangle + mg\langle 0,0,-1\rangle = \mathbf{0}$$

$$\left\langle \sqrt{\frac{2}{3}}T_{\text{DA}} - \sqrt{\frac{2}{3}}T_{\text{DB}} - \frac{2}{3}T_{\text{DC}},\ -\frac{1}{\sqrt{6}}T_{\text{DA}} - \frac{1}{\sqrt{6}}T_{\text{DB}} + \frac{2}{3}T_{\text{DC}},\ \frac{1}{\sqrt{6}}T_{\text{DA}} + \frac{1}{\sqrt{6}}T_{\text{DB}} + \frac{1}{3}T_{\text{DC}} - mg \right\rangle = \langle 0,0,0\rangle$$

Match the components to get a system of equations.

$$\left.\begin{aligned} \sqrt{\frac{2}{3}}T_{\text{DA}} - \sqrt{\frac{2}{3}}T_{\text{DB}} - \frac{2}{3}T_{\text{DC}} &= 0 \\ -\frac{1}{\sqrt{6}}T_{\text{DA}} - \frac{1}{\sqrt{6}}T_{\text{DB}} + \frac{2}{3}T_{\text{DC}} &= 0 \\ \frac{1}{\sqrt{6}}T_{\text{DA}} + \frac{1}{\sqrt{6}}T_{\text{DB}} + \frac{1}{3}T_{\text{DC}} - mg &= 0 \end{aligned}\right\}$$

Solving it and plugging in $m = 130$ kg and $g = 9.81$ m/s^2 yields

$$T_{\text{DA}} = \sqrt{\frac{3}{2}}mg \approx 1.56\times 10^3 \text{ N}$$

$$T_{\text{DB}} = \frac{1}{\sqrt{6}}mg \approx 521 \text{ N}$$

$$T_{\text{DC}} = mg \approx 1.28\times 10^3 \text{ N}.$$