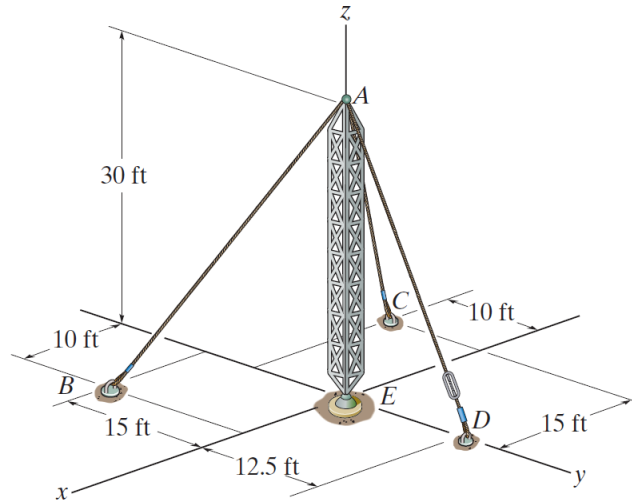


Problem 3-64

If cable AD is tightened by a turnbuckle and develops a tension of 1300 lb, determine the tension developed in cables AB and AC and the force developed along the antenna tower AE at point A .



Prob. 3-64

Solution

Write position vectors to points A , B , C , and D .

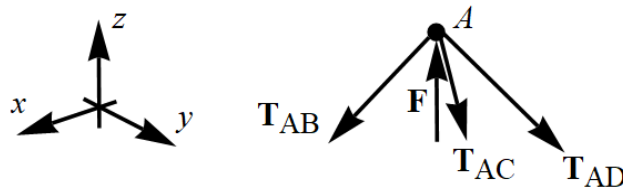
$$\mathbf{r}_A = \langle 0, 0, 30 \rangle \text{ ft}$$

$$\mathbf{r}_B = \langle 10, -15, 0 \rangle \text{ ft}$$

$$\mathbf{r}_C = \langle -15, -10, 0 \rangle \text{ ft}$$

$$\mathbf{r}_D = \langle 0, 12.5, 0 \rangle \text{ ft}$$

Draw a free-body diagram for point A .



In order for the system to be in equilibrium, the sum of the forces must be zero.

$$\mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + \mathbf{F} = \mathbf{0}$$

$$T_{AB}\hat{\mathbf{u}}_{AB} + T_{AC}\hat{\mathbf{u}}_{AC} + T_{AD}\hat{\mathbf{u}}_{AD} + F\langle 0, 0, 1 \rangle = \mathbf{0}$$

$$T_{AB}\frac{\mathbf{r}_B - \mathbf{r}_A}{|\mathbf{r}_B - \mathbf{r}_A|} + T_{AC}\frac{\mathbf{r}_C - \mathbf{r}_A}{|\mathbf{r}_C - \mathbf{r}_A|} + T_{AD}\frac{\mathbf{r}_D - \mathbf{r}_A}{|\mathbf{r}_D - \mathbf{r}_A|} + F\langle 0, 0, 1 \rangle = \mathbf{0}$$

Write out the unit vectors and simplify the left side.

$$T_{AB} \frac{\langle 10 - 0, -15 - 0, 0 - 30 \rangle}{\sqrt{(10 - 0)^2 + (-15 - 0)^2 + (0 - 30)^2}} + T_{AC} \frac{\langle -15 - 0, -10 - 0, 0 - 30 \rangle}{\sqrt{(-15 - 0)^2 + (-10 - 0)^2 + (0 - 30)^2}} \\ + T_{AD} \frac{\langle 0 - 0, 12.5 - 0, 0 - 30 \rangle}{\sqrt{(0 - 0)^2 + (12.5 - 0)^2 + (0 - 30)^2}} + F \langle 0, 0, 1 \rangle = \mathbf{0}$$

$$T_{AB} \left\langle \frac{2}{7}, -\frac{3}{7}, -\frac{6}{7} \right\rangle + T_{AC} \left\langle -\frac{3}{7}, -\frac{2}{7}, -\frac{6}{7} \right\rangle + T_{AD} \left\langle 0, \frac{5}{13}, -\frac{12}{13} \right\rangle + F \langle 0, 0, 1 \rangle = \mathbf{0}$$

$$\left\langle \frac{2}{7}T_{AB} - \frac{3}{7}T_{AC}, -\frac{3}{7}T_{AB} - \frac{2}{7}T_{AC} + \frac{5}{13}T_{AD}, -\frac{6}{7}T_{AB} - \frac{6}{7}T_{AC} - \frac{12}{13}T_{AD} + F \right\rangle = \langle 0, 0, 0 \rangle$$

Match the components to get a system of equations.

$$\left. \begin{aligned} \frac{2}{7}T_{AB} - \frac{3}{7}T_{AC} &= 0 \\ -\frac{3}{7}T_{AB} - \frac{2}{7}T_{AC} + \frac{5}{13}T_{AD} &= 0 \\ -\frac{6}{7}T_{AB} - \frac{6}{7}T_{AC} - \frac{12}{13}T_{AD} + F &= 0 \end{aligned} \right\}$$

Solve it for T_{AB} , T_{AC} , and F and then plug in $T_{AD} = 1300$ lb.

$$T_{AB} = \frac{105}{169}T_{AD} \approx 808 \text{ lb}$$

$$T_{AC} = \frac{70}{169}T_{AD} \approx 538 \text{ lb}$$

$$F = \frac{306}{169}T_{AD} \approx 2.35 \times 10^3 \text{ lb}$$