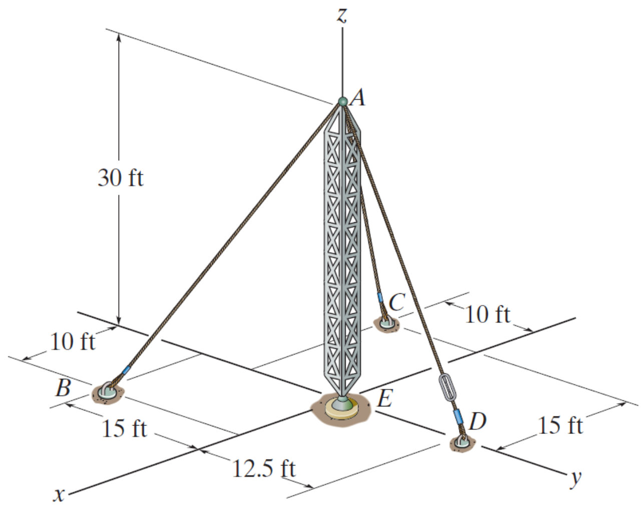
Problem 3-65

If the tension developed in either cable AB or AC can not exceed 1000 lb, determine the maximum tension that can be developed in cable AD when it is tightened by the turnbuckle. Also, what is the force developed along the antenna tower at point A?



Prob. 3–65

Solution

Write position vectors to points A, B, C, and D.

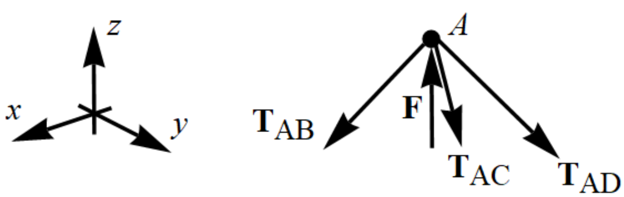
$$\mathbf{r}_A = \langle 0, 0, 30 \rangle \text{ ft}$$

$$\mathbf{r}_B = \langle 10, -15, 0 \rangle \text{ ft}$$

$$\mathbf{r}_C = \langle -15, -10, 0 \rangle \text{ ft}$$

$$\mathbf{r}_D = \langle 0, 12.5, 0 \rangle \text{ ft}$$

Draw a free-body diagram for point A.



In order for the system to be in equilibrium, the sum of the forces must be zero.

$$\mathbf{T}_{\text{AB}} + \mathbf{T}_{\text{AC}} + \mathbf{T}_{\text{AD}} + \mathbf{F} = \mathbf{0}$$

$$T_{\text{AB}}\hat{\mathbf{u}}_{\text{AB}} + T_{\text{AC}}\hat{\mathbf{u}}_{\text{AC}} + T_{\text{AD}}\hat{\mathbf{u}}_{\text{AD}} + F\langle 0, 0, 1 \rangle = \mathbf{0}$$

$$T_{\text{AB}}\frac{\mathbf{r}_B - \mathbf{r}_A}{|\mathbf{r}_B - \mathbf{r}_A|} + T_{\text{AC}}\frac{\mathbf{r}_C - \mathbf{r}_A}{|\mathbf{r}_C - \mathbf{r}_A|} + T_{\text{AD}}\frac{\mathbf{r}_D - \mathbf{r}_A}{|\mathbf{r}_D - \mathbf{r}_A|} + F\langle 0, 0, 1 \rangle = \mathbf{0}$$

Write out the unit vectors and simplify the left side.

$$T_{\text{AB}}\frac{\langle 10-0,-15-0,0-30\rangle}{\sqrt{(10-0)^2+(-15-0)^2+(0-30)^2}}+T_{\text{AC}}\frac{\langle -15-0,-10-0,0-30\rangle}{\sqrt{(-15-0)^2+(-10-0)^2+(0-30)^2}}$$
$$+T_{\text{AD}}\frac{\langle 0-0,12.5-0,0-30\rangle}{\sqrt{(0-0)^2+(12.5-0)^2+(0-30)^2}}+F\langle 0,0,1\rangle=\mathbf{0}$$

$$T_{\text{AB}}\left\langle \frac{2}{7},-\frac{3}{7},-\frac{6}{7}\right\rangle+T_{\text{AC}}\left\langle -\frac{3}{7},-\frac{2}{7},-\frac{6}{7}\right\rangle+T_{\text{AD}}\left\langle 0,\frac{5}{13},-\frac{12}{13}\right\rangle+F\langle 0,0,1\rangle=\mathbf{0}$$

$$\left\langle \frac{2}{7}T_{\text{AB}}-\frac{3}{7}T_{\text{AC}},\ -\frac{3}{7}T_{\text{AB}}-\frac{2}{7}T_{\text{AC}}+\frac{5}{13}T_{\text{AD}},\ -\frac{6}{7}T_{\text{AB}}-\frac{6}{7}T_{\text{AC}}-\frac{12}{13}T_{\text{AD}}+F\right\rangle=\langle 0,0,0\rangle$$

Match the components to get a system of equations.

$$\left.\begin{aligned}\frac{2}{7}T_{\text{AB}}-\frac{3}{7}T_{\text{AC}}&=0\\ -\frac{3}{7}T_{\text{AB}}-\frac{2}{7}T_{\text{AC}}+\frac{5}{13}T_{\text{AD}}&=0\\ -\frac{6}{7}T_{\text{AB}}-\frac{6}{7}T_{\text{AC}}-\frac{12}{13}T_{\text{AD}}+F&=0\end{aligned}\right\}$$

In the previous problem it was found that cable AB has a higher tension than that in cable AC, so set $T_{\text{AB}}=1000$ lb and solve the system for T_{AC}, T_{AD}, and F.

$$\begin{aligned}T_{\text{AC}}&=\frac{2000}{3}\text{ lb}\approx 667\text{ lb}\\ T_{\text{AD}}&=\frac{33\,800}{21}\text{ lb}\approx 1.61\times 10^3\text{ lb}\\ F&=\frac{20\,400}{7}\text{ lb}\approx 2.91\times 10^3\text{ lb}\end{aligned}$$