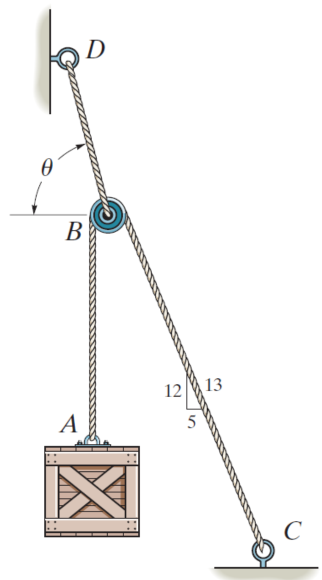
Problem 3-8

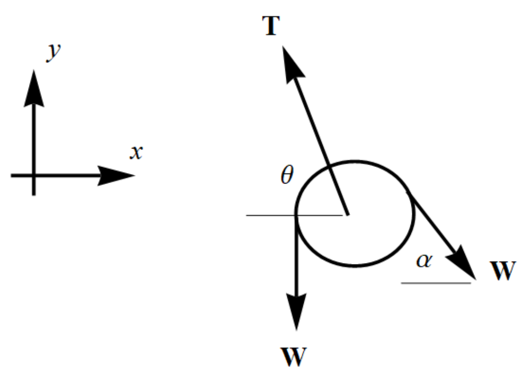
The cords ABC and BD can each support a maximum load of 100 lb. Determine the maximum weight of the crate, and the angle θ for equilibrium.



Prob. 3–8

Solution

Draw a free-body diagram for the pulley, noting that because the pulley is frictionless, the tension is the same everywhere in the cable.



Use the provided triangle to determine α.

$$\tan\alpha = \frac{12}{5} \quad \rightarrow \quad \alpha = \tan^{-1}\left(\frac{12}{5}\right) \approx 67.4^\circ$$

In order for the pulley to be in equilibrium, the sum of the forces in each direction must be zero.

$$\sum F_x = 0: \qquad W\cos\alpha - T\cos\theta = 0$$

$$\sum F_y = 0: \qquad -W - W\sin\alpha + T\sin\theta = 0$$

Since cable BD is supporting two times the weight of the crate, it has the highest load: Set $T = 100$ lb and solve the system of equations for W and θ.

$$W\cos\alpha - 100\cos\theta = 0 \tag{1}$$

$$-W - W\sin\alpha + 100\sin\theta = 0 \tag{2}$$

Solve for the terms with θ.

$$100\cos\theta = W\cos\alpha$$

$$100\sin\theta = W(1+\sin\alpha)$$

Square both sides of each equation and then add them respectively to eliminate θ.

$$100^2(\cos^2\theta + \sin^2\theta) = W^2\cos^2\alpha + W^2(1+\sin\alpha)^2$$

$$100^2(1) = W^2\cos^2\alpha + W^2(1+\sin\alpha)^2$$

$$W^2 = \frac{100^2}{\cos^2\alpha + (1+\sin\alpha)^2}$$

$$W = \sqrt{\frac{100^2}{\cos^2\alpha + (1+\sin\alpha)^2}}\ \text{lb}$$

$$\approx 51.0\ \text{lb}$$

Substitute this value for W into equation (1) to determine θ.

$$W\cos\alpha - 100\cos\theta = 0 \quad\rightarrow\quad \cos\theta = \frac{W\cos\alpha}{100} \quad\rightarrow\quad \theta \approx 78.7°$$