Vector Identity 14

$$\nabla \cdot [\mathbf{n} f(r)] = \frac{2}{r} f + \frac{\partial f}{\partial r}$$

Proof

Use the formula for the divergence in spherical coordinates (r, ϕ, θ), where θ is the angle from the polar axis.

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2} \frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta}(\sin\theta\, A_\theta) + \frac{1}{r \sin\theta} \frac{\partial A_\phi}{\partial \phi}$$

Since $\mathbf{n}$ is a radial unit vector, the last two terms are zero.

$$\begin{aligned}
\nabla \cdot [\mathbf{n} f(r)] &= \frac{1}{r^2} \frac{\partial}{\partial r}[r^2 f(r)] + \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta}[\sin\theta\,(0)] + \frac{1}{r \sin\theta} \frac{\partial (0)}{\partial \phi} \\
&= \frac{1}{r^2} \frac{d}{dr}[r^2 f(r)] \\
&= \frac{1}{r^2} \left[2r f(r) + r^2 \frac{df}{dr}\right] \\
&= \frac{2}{r} f + \frac{df}{dr}
\end{aligned}$$