Vector Identity 15

$$\nabla \times [\mathbf{n} f(r)] = 0$$

Proof

Use the formula for the curl in spherical coordinates (r, ϕ, θ), where θ is the angle from the polar axis.

$$\nabla \times \mathbf{A} = \hat{\mathbf{r}} \left\{ \frac{1}{r \sin\theta} \left[\frac{\partial}{\partial\theta}(\sin\theta \, A_\phi) - \frac{\partial A_\theta}{\partial\phi} \right] \right\} + \hat{\boldsymbol{\theta}} \left[\frac{1}{r \sin\theta} \frac{\partial A_r}{\partial\phi} - \frac{1}{r} \frac{\partial}{\partial r}(r A_\phi) \right] + \hat{\boldsymbol{\phi}} \frac{1}{r} \left[\frac{\partial}{\partial r}(r A_\theta) - \frac{\partial A_r}{\partial\theta} \right]$$

Since $\mathbf{n}$ is a radial unit vector, $A_r = f(r)$ and $A_\phi = 0$ and $A_\theta = 0$.

$$\nabla \times [\mathbf{n} f(r)] = \hat{\boldsymbol{\theta}} \frac{1}{r \sin\theta} \frac{\partial}{\partial\phi}[f(r)] - \hat{\boldsymbol{\phi}} \frac{1}{r} \frac{\partial}{\partial\theta}[f(r)]$$

And since f is only a function of r, the right side is zero.

$$\nabla \times [\mathbf{n} f(r)] = \mathbf{0}$$