

Vector Identity 2

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$$

Proof

Let $\mathbf{A} = \mathbf{a}$, $\mathbf{B} = \mathbf{b}$, and $\mathbf{C} = \mathbf{c}$.

$$\begin{aligned}
\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) &= \left(\sum_{i=1}^3 \delta_i A_i \right) \times \left[\left(\sum_{j=1}^3 \delta_j B_j \right) \times \left(\sum_{k=1}^3 \delta_k C_k \right) \right] \\
&= \left(\sum_{i=1}^3 \delta_i A_i \right) \times \left[\sum_{j=1}^3 \sum_{k=1}^3 (\delta_j \times \delta_k) B_j C_k \right] \\
&= \left(\sum_{i=1}^3 \delta_i A_i \right) \times \left(\sum_{j=1}^3 \sum_{k=1}^3 \sum_{l=1}^3 \delta_l \varepsilon_{ljk} B_j C_k \right) \\
&= \sum_{i=1}^3 \sum_{j=1}^3 \sum_{k=1}^3 \sum_{l=1}^3 (\delta_i \times \delta_l) \varepsilon_{ljk} A_i B_j C_k \\
&= \sum_{i=1}^3 \sum_{j=1}^3 \sum_{k=1}^3 \sum_{l=1}^3 \sum_{m=1}^3 \delta_m \varepsilon_{ilm} \varepsilon_{ljk} A_i B_j C_k \\
&= \sum_{i=1}^3 \sum_{j=1}^3 \sum_{k=1}^3 \sum_{l=1}^3 \sum_{m=1}^3 \delta_m \varepsilon_{lmi} \varepsilon_{ljk} A_i B_j C_k \\
&= \sum_{i=1}^3 \sum_{j=1}^3 \sum_{k=1}^3 \sum_{m=1}^3 \delta_m (\delta_{mj} \delta_{ik} - \delta_{mk} \delta_{ij}) A_i B_j C_k \\
&= \sum_{i=1}^3 \sum_{j=1}^3 \sum_{k=1}^3 \sum_{m=1}^3 (\delta_m \delta_{mj} \delta_{ik} A_i B_j C_k - \delta_m \delta_{mk} \delta_{ij} A_i B_j C_k) \\
&= \sum_{i=1}^3 \sum_{j=1}^3 \sum_{k=1}^3 \sum_{m=1}^3 \delta_m \delta_{mj} \delta_{ik} A_i B_j C_k - \sum_{i=1}^3 \sum_{j=1}^3 \sum_{k=1}^3 \sum_{m=1}^3 \delta_m \delta_{mk} \delta_{ij} A_i B_j C_k \\
&= \sum_{i=1}^3 \sum_{j=1}^3 \sum_{k=1}^3 \delta_j \delta_{ik} A_i B_j C_k - \sum_{i=1}^3 \sum_{j=1}^3 \sum_{k=1}^3 \delta_k \delta_{ij} A_i B_j C_k \\
&= \sum_{i=1}^3 \sum_{j=1}^3 \delta_j A_i B_j C_i - \sum_{j=1}^3 \sum_{k=1}^3 \delta_k A_j B_j C_k \\
&= \left(\sum_{i=1}^3 A_i C_i \right) \sum_{j=1}^3 \delta_j B_j - \left(\sum_{j=1}^3 A_j B_j \right) \sum_{k=1}^3 \delta_k C_k \\
&= (\mathbf{A} \cdot \mathbf{C})\mathbf{B} - (\mathbf{A} \cdot \mathbf{B})\mathbf{C}
\end{aligned}$$