Vector Identity 8

$$\nabla \times (\psi \mathbf{a}) = \nabla \psi \times \mathbf{a} + \psi \nabla \times \mathbf{a}$$

Proof

Let $f = \psi$ and let $\mathbf{A} = \mathbf{a}$.

$$\begin{aligned}
\nabla \times (f\mathbf{A}) &= \left(\sum_{i=1}^{3} \boldsymbol{\delta}_i \frac{\partial}{\partial x_i}\right) \times \left[f \left(\sum_{j=1}^{3} \boldsymbol{\delta}_j A_j \right) \right] \\
&= \left(\sum_{i=1}^{3} \boldsymbol{\delta}_i \frac{\partial}{\partial x_i}\right) \times \left(\sum_{j=1}^{3} \boldsymbol{\delta}_j A_j f \right) \\
&= \sum_{i=1}^{3} \sum_{j=1}^{3} (\boldsymbol{\delta}_i \times \boldsymbol{\delta}_j) \frac{\partial}{\partial x_i} A_j f \\
&= \sum_{i=1}^{3} \sum_{j=1}^{3} (\boldsymbol{\delta}_i \times \boldsymbol{\delta}_j) \left(A_j \frac{\partial f}{\partial x_i} + \frac{\partial A_j}{\partial x_i} f \right) \\
&= \sum_{i=1}^{3} \sum_{j=1}^{3} (\boldsymbol{\delta}_i \times \boldsymbol{\delta}_j) A_j \frac{\partial f}{\partial x_i} + \sum_{i=1}^{3} \sum_{j=1}^{3} (\boldsymbol{\delta}_i \times \boldsymbol{\delta}_j) \frac{\partial A_j}{\partial x_i} f \\
&= \sum_{i=1}^{3} \sum_{j=1}^{3} (\boldsymbol{\delta}_i \times \boldsymbol{\delta}_j) A_j \frac{\partial f}{\partial x_i} + f \left[\sum_{i=1}^{3} \sum_{j=1}^{3} (\boldsymbol{\delta}_i \times \boldsymbol{\delta}_j) \frac{\partial A_j}{\partial x_i} \right] \\
&= \left(\sum_{i=1}^{3} \boldsymbol{\delta}_i \frac{\partial f}{\partial x_i}\right) \times \left(\sum_{j=1}^{3} \boldsymbol{\delta}_j A_j \right) + f \left[\left(\sum_{i=1}^{3} \boldsymbol{\delta}_i \frac{\partial}{\partial x_i}\right) \times \left(\sum_{j=1}^{3} \boldsymbol{\delta}_j A_j \right) \right] \\
&= \nabla f \times \mathbf{A} + f \nabla \times \mathbf{A}
\end{aligned}$$