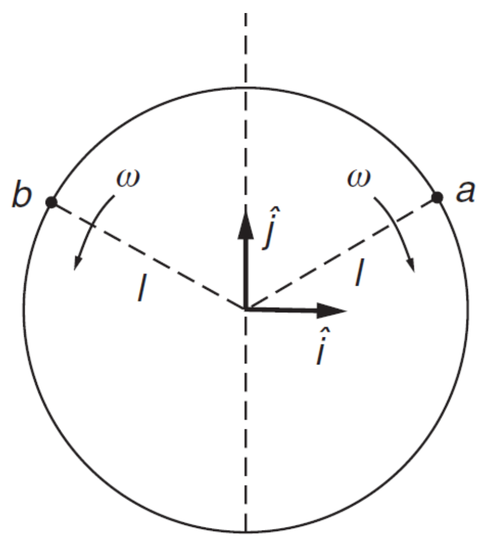
Problem 1.19

Relative velocity
By *relative velocity* we mean velocity with respect to a specified coordinate system. (The term velocity, alone, is understood to be relative to the observer's coordinate system.)

(a) A point is observed to have velocity $\mathbf{v}_A$ relative to coordinate system A. What is its velocity relative to coordinate system B, which is displaced from system A by distance $\mathbf{R}$? ($\mathbf{R}$ can change in time.)

(b) Particles a and b move in opposite directions around a circle with angular speed ω, as shown. At $t = 0$ they are both at the point $\mathbf{r} = l\hat{\mathbf{j}}$, where l is the radius of the circle.
Find the velocity of a relative to b.



Solution

Part (a)

The position of a point with respect to coordinate system A is

$$\mathbf{x}_A = \mathbf{R} + \mathbf{x}_B.$$

Differentiate both sides with respect to t.

$$\frac{d}{dt}\mathbf{x}_A = \frac{d}{dt}\mathbf{R} + \frac{d}{dt}\mathbf{x}_B$$

$$\mathbf{v}_A = \frac{d\mathbf{R}}{dt} + \mathbf{v}_B$$

Therefore,

$$\mathbf{v}_B = \mathbf{v}_A - \frac{d\mathbf{R}}{dt}.$$

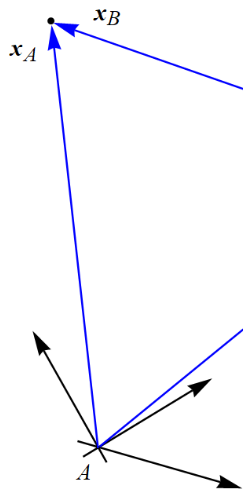
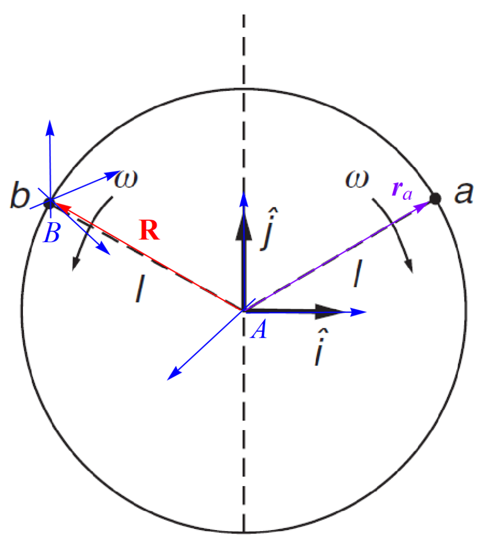


Figure 1: This figure illustrates the position of a point with respect to two coordinate systems, A and B, the latter being displaced from the former by $\mathbf{R} = \mathbf{R}(t)$.

Part (b)

The aim here is to apply the result of part (a). We want to find the velocity of particle a from the perspective of particle b, so coordinate system B is set up where particle b is. Coordinate system A is set up at the center of the circle for convenience. Consequently, $\mathbf{R}$ is the position vector of particle b with respect to coordinate system A, and $\mathbf{v}_A$ is the velocity of particle a with respect to coordinate system A.

$$\mathbf{r}_a(t) = \langle l \sin \omega t, l \cos \omega t \rangle$$
$$\mathbf{R}(t) = \langle -l \sin \omega t, l \cos \omega t \rangle$$



Therefore, the velocity of particle a from the perspective of particle b is

$$\mathbf{v}_B = \mathbf{v}_A - \frac{d\mathbf{R}}{dt} = \frac{d\mathbf{r}_a}{dt} - \frac{d\mathbf{R}}{dt} = \langle l\omega \cos \omega t, -l\omega \sin \omega t \rangle - \langle -l\omega \cos \omega t, -l\omega \sin \omega t \rangle = \langle 2l\omega \cos \omega t, 0 \rangle.$$