Problem 1.9

*Perpendicular unit vector**
Find a unit vector perpendicular to $\mathbf{A} = (\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}})$ and $\mathbf{B} = (2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}})$.

Solution

Taking the cross product of $\mathbf{A}$ and $\mathbf{B}$ will give us another vector that is perpendicular to both of them.

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 1 & -1 \\ 2 & 1 & -3 \end{vmatrix} = \hat{\mathbf{i}}[(1)(-3) - (-1)(1)] - \hat{\mathbf{j}}[(1)(-3) - (-1)(2)] + \hat{\mathbf{k}}[(1)(1) - (1)(2)]$$

$$\mathbf{A} \times \mathbf{B} = -2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$$

Let's call this new vector $\mathbf{n}$. Because we want a perpendicular unit vector, we have to divide $\mathbf{n}$ by its magnitude. $\hat{\mathbf{n}}$ is the unit vector in the direction of $\mathbf{n}$.

$$\hat{\mathbf{n}} = \frac{\mathbf{n}}{|\mathbf{n}|} = \frac{-2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}}{\sqrt{(-2)^2 + 1^2 + (-1)^2}}$$

Therefore, a perpendicular unit vector is

$$\hat{\mathbf{n}} = \frac{1}{\sqrt{6}}(-2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}).$$

Note that $-\hat{\mathbf{n}}$, the unit vector antiparallel to $\hat{\mathbf{n}}$, is also perpendicular to $\mathbf{A}$ and $\mathbf{B}$.