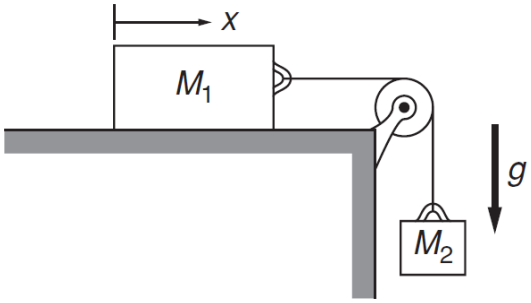
Problem 2.2

Two blocks and string
The two blocks M_1 and M_2 shown in the sketch are connected by a string of negligible mass. If the system is released from rest, find how far block M_1 slides in time t. Neglect friction.

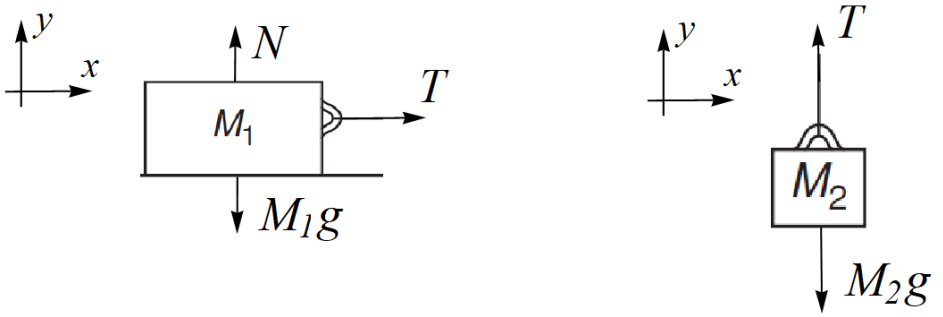


Solution

The strategy here is to apply Newton's second law to determine the acceleration a of block M_1 and then to use the kinematic formula,

$$x = x_0 + v_0 t + \frac{1}{2}at^2,$$

to find how far it slides in time t. We assume that the pulley is frictionless so that the tension T in each part of the string is the same. Draw the free-body diagram of each block.



Newton's second law states that the sum of the forces is equal to mass times acceleration.

$$\sum \mathbf{F} = m\mathbf{a}.$$

This vector equation represents the following two scalar equations in the chosen coordinate system.

$$\sum F_x = ma_x$$
$$\sum F_y = ma_y$$

Let a denote the acceleration of block M_1. Because block M_1 and block M_2 are attached to the same string, they have the same acceleration. Block M_2 moves in the negative y-direction,

though, so it has acceleration $-a$. Apply Newton's second law to each block.

Block M_1	Block M_2
$\sum F_x = T = M_1 a$	$\sum F_x = 0 = M_2(0)$
$\sum F_y = N - M_1 g = M_1(0)$	$\sum F_y = T - M_2 g = M_2(-a)$

Solve the system of equations for a, the variable of interest, by eliminating T.

$$T - M_2 g = -M_2 a$$

$$M_1 a - M_2 g = -M_2 a$$

$$M_1 a + M_2 a = M_2 g$$

$$a = \frac{M_2}{M_1 + M_2} g$$

Since the acceleration is constant, we can use the kinematic formula,

$$x = x_0 + v_0 t + \frac{1}{2} a t^2,$$

to find how far it moves in time t. Block M_1 starts from rest, so $v_0 = 0$.

$$x = x_0 + \frac{1}{2} a t^2,$$

Bring x_0 to the left side and substitute the formula for a.

$$x - x_0 = \frac{1}{2} \frac{M_2}{M_1 + M_2} g t^2$$

Therefore, the displacement of block M_1 in time t, is

$$\Delta x = \frac{1}{2} \frac{M_2}{M_1 + M_2} g t^2.$$