Problem 1.1

*Vector algebra 1**
Given two vectors $\mathbf{A} = (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 7\hat{\mathbf{k}})$ and $\mathbf{B} = (5\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ find:
(a) $\mathbf{A} + \mathbf{B}$; (b) $\mathbf{A} - \mathbf{B}$; (c) $\mathbf{A} \cdot \mathbf{B}$; (d) $\mathbf{A} \times \mathbf{B}$.

Solution

Part (a)

To find $\mathbf{A} + \mathbf{B}$, we add their respective components together.

$$\mathbf{A} + \mathbf{B} = (2+5)\hat{\mathbf{i}} + (-3+1)\hat{\mathbf{j}} + (7+2)\hat{\mathbf{k}} = 7\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 9\hat{\mathbf{k}}$$

Part (b)

To find $\mathbf{A} - \mathbf{B}$, we subtract the components of $\mathbf{B}$ from the respective components of $\mathbf{A}$.

$$\mathbf{A} - \mathbf{B} = (2-5)\hat{\mathbf{i}} + (-3-1)\hat{\mathbf{j}} + (7-2)\hat{\mathbf{k}} = -3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$$

Part (c)

To find $\mathbf{A} \cdot \mathbf{B}$, we multiply their respective components together and add them. Note that $\mathbf{A} \cdot \mathbf{B}$ is a scalar, not a vector.

$$\mathbf{A} \cdot \mathbf{B} = (2)(5) + (-3)(1) + (7)(2) = 21$$

Part (d)

To find $\mathbf{A} \times \mathbf{B}$, we evaluate a 3×3 determinant, where the unit vectors are in the first row, the components of $\mathbf{A}$ are in the second row, and the components of $\mathbf{B}$ are in the third row.

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & -3 & 7 \\ 5 & 1 & 2 \end{vmatrix} = \begin{vmatrix} -3 & 7 \\ 1 & 2 \end{vmatrix} \hat{\mathbf{i}} - \begin{vmatrix} 2 & 7 \\ 5 & 2 \end{vmatrix} \hat{\mathbf{j}} + \begin{vmatrix} 2 & -3 \\ 5 & 1 \end{vmatrix} \hat{\mathbf{k}} = -13\hat{\mathbf{i}} + 31\hat{\mathbf{j}} + 17\hat{\mathbf{k}}$$