

Exercise 13

Solve the systems in Exercises 11–14.

$$\begin{aligned}x_1 - 3x_3 &= 8 \\2x_1 + 2x_2 + 9x_3 &= 7 \\x_2 + 5x_3 &= -2\end{aligned}$$

Solution

Write the augmented matrix corresponding to this system of equations.

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 2 & 2 & 9 & 7 \\ 0 & 1 & 5 & -2 \end{array} \right]$$

To make the middle-left entry 0, multiply the first row by -2 and add it to the second row.

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 2 & 15 & -9 \\ 0 & 1 & 5 & -2 \end{array} \right]$$

Switch the second row with the third row.

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 2 & 15 & -9 \end{array} \right]$$

To make the bottom-middle entry 0, multiply the second row by -2 and add it to the third row.

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 5 & -5 \end{array} \right]$$

Divide the third row by 5.

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 1 & -1 \end{array} \right]$$

Multiply the third row by -5 and add it to the second row.

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array} \right]$$

Multiply the third row by 3 and add it to the first row.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array} \right]$$

Therefore,

$$\begin{aligned}x_1 &= 5 \\x_2 &= 3 \\x_3 &= -1\end{aligned}$$

which means the solution set is $\{5, 3, -1\}$.