

Exercise 14

Solve the systems in Exercises 11–14.

$$\begin{aligned}x_1 - 3x_2 &= 5 \\ -x_1 + x_2 + 5x_3 &= 2 \\ x_2 + x_3 &= 0\end{aligned}$$

Solution

Write the augmented matrix corresponding to this system of equations.

$$\left[\begin{array}{ccc|c} 1 & -3 & 0 & 5 \\ -1 & 1 & 5 & 2 \\ 0 & 1 & 1 & 0 \end{array} \right]$$

Switch the third row with the second row.

$$\left[\begin{array}{ccc|c} 1 & -3 & 0 & 5 \\ 0 & 1 & 1 & 0 \\ -1 & 1 & 5 & 2 \end{array} \right]$$

Add the first row to the third row.

$$\left[\begin{array}{ccc|c} 1 & -3 & 0 & 5 \\ 0 & 1 & 1 & 0 \\ 0 & -2 & 5 & 7 \end{array} \right]$$

Multiply the second row by 2 and add it to the third row.

$$\left[\begin{array}{ccc|c} 1 & -3 & 0 & 5 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 7 & 7 \end{array} \right]$$

Divide the third row by 7.

$$\left[\begin{array}{ccc|c} 1 & -3 & 0 & 5 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right]$$

Multiply the third row by -1 and add it to the second row.

$$\left[\begin{array}{ccc|c} 1 & -3 & 0 & 5 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array} \right]$$

Multiply the second row by 3 and add it to the first row.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array} \right]$$

Therefore,

$$\begin{aligned}x_1 &= 2 \\ x_2 &= -1 \\ x_3 &= 1\end{aligned}$$

which means the solution set is $\{2, -1, 1\}$.