

Exercise 16

Determine if the systems in Exercises 15 and 16 are consistent. Do not completely solve the systems.

$$\begin{array}{rcl} x_1 & -2x_4 & = -3 \\ & 2x_2 + 2x_3 & = 0 \\ & x_3 + 3x_4 & = 1 \\ -2x_1 + 3x_2 + 2x_3 + x_4 & & = 5 \end{array}$$

Solution

Write the augmented matrix corresponding to this system of equations.

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -3 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ -2 & 3 & 2 & 1 & 5 \end{array} \right]$$

The aim is to write this in triangular form. To make the bottom left entry 0, multiply the first row by 2 and add it to the fourth row.

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -3 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 3 & 2 & -3 & -1 \end{array} \right]$$

Divide the second row by 2.

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -3 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 3 & 2 & -3 & -1 \end{array} \right]$$

Multiply the second row by -3 and add it to the fourth row.

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -3 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & -1 & -3 & -1 \end{array} \right]$$

Add the third row to the fourth row.

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -3 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

Since a solution for x_1 , x_2 , x_3 , and x_4 exists, the system is consistent.