

**Exercise 17**

Do the three lines  $x_1 - 4x_2 = 1$ ,  $2x_1 - x_2 = -3$ , and  $-x_1 - 3x_2 = 4$  have a common point of intersection? Explain.

**Solution**

In order for the lines to have a common point of intersection, there has to be a solution for  $x_1$  and  $x_2$  that satisfies the following system.

$$\begin{aligned}x_1 - 4x_2 &= 1 \\2x_1 - x_2 &= -3 \\-x_1 - 3x_2 &= 4\end{aligned}$$

Write the augmented matrix corresponding to this system of equations.

$$\left[ \begin{array}{cc|c} 1 & -4 & 1 \\ 2 & -1 & -3 \\ -1 & -3 & 4 \end{array} \right]$$

Multiply the first row by  $-2$  and add it to the second row.

$$\left[ \begin{array}{cc|c} 1 & -4 & 1 \\ 0 & 7 & -5 \\ -1 & -3 & 4 \end{array} \right]$$

Add the first row to the third row.

$$\left[ \begin{array}{cc|c} 1 & -4 & 1 \\ 0 & 7 & -5 \\ 0 & -7 & 5 \end{array} \right]$$

Add the second row to the third row.

$$\left[ \begin{array}{cc|c} 1 & -4 & 1 \\ 0 & 7 & -5 \\ 0 & 0 & 0 \end{array} \right]$$

Divide the second row by 7.

$$\left[ \begin{array}{cc|c} 1 & -4 & 1 \\ 0 & 1 & -\frac{5}{7} \\ 0 & 0 & 0 \end{array} \right]$$

Multiply the second row by 4 and add it to the first row.

$$\left[ \begin{array}{cc|c} 1 & 0 & -\frac{13}{7} \\ 0 & 1 & -\frac{5}{7} \\ 0 & 0 & 0 \end{array} \right]$$

Therefore,  $x_1 - 4x_2 = 1$ ,  $2x_1 - x_2 = -3$ , and  $-x_1 - 3x_2 = 4$  do have a common point of intersection at  $x_1 = -13/7$  and  $x_2 = -5/7$ . This could've been noticed immediately by the fact that  $-x_1 - 3x_2 = 4$  is obtained by subtracting the respective sides of  $2x_1 - x_2 = -3$  from those of  $x_1 - 4x_2 = 1$ .