Exercise 18

Do the three planes $x_1 + 2x_2 + x_3 = 4$, $x_2 - x_3 = 1$, and $x_1 + 3x_2 = 0$ have at least one common point of intersection? Explain.

Solution

In order for the planes to have a common point of intersection, there has to be a solution for x_1 and x_2 and x_3 that satisfies the following system.

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 4 \\ x_2 - x_3 &= 1 \\ x_1 + 3x_2 &= 0 \end{aligned}$$

Write the augmented matrix corresponding to this system of equations.

$$\left[\begin{array}{rrr|r} 1 & 2 & 1 & 4 \\ 0 & 1 & -1 & 1 \\ 1 & 3 & 0 & 0 \end{array}\right]$$

Multiply the first row by -1 and add it to the third row.

$$\left[\begin{array}{rrr|r} 1 & 2 & 1 & 4 \\ 0 & 1 & -1 & 1 \\ 0 & 1 & -1 & -4 \end{array}\right]$$

Multiply the second row by -1 and add it to the third row.

$$\left[\begin{array}{rrr|r} 1 & 2 & 1 & 4 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & -5 \end{array}\right]$$

The last row means that $0x_1 + 0x_2 + 0x_3 = 5$. No values of x_1, x_2, and x_3 can make this a true statement. That means there's no solution to the system. Therefore, the three planes $x_1 + 2x_2 + x_3 = 4$, $x_2 - x_3 = 1$, and $x_1 + 3x_2 = 0$ do not have a common point of intersection.