

Exercise 26

Construct three different augmented matrices for linear systems whose solution set is $x_1 = -2$, $x_2 = 1$, $x_3 = 0$.

Solution

Three systems of equations with $x_1 = -2$, $x_2 = 1$, and $x_3 = 0$ are as follows.

$$\left. \begin{array}{l} x_1 + 2x_2 = 0 \\ 3x_1 + 6x_2 + x_3 = 0 \\ -2x_1 + 4x_2 + 5x_3 = 8 \end{array} \right\} \Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 3 & 6 & 1 & 0 \\ -2 & 4 & 5 & 8 \end{array} \right]$$
$$\left. \begin{array}{l} x_1 + x_2 + x_3 = -1 \\ -x_1 + 3x_2 - 6x_3 = 5 \\ -2x_1 + 4x_2 + x_3 = 8 \end{array} \right\} \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & -1 \\ -1 & 3 & -6 & 5 \\ -2 & 4 & 1 & 8 \end{array} \right]$$
$$\left. \begin{array}{l} x_1 - 4x_2 + 7x_3 = -6 \\ -5x_1 + x_2 + 2x_3 = 11 \\ 2x_1 + 3x_2 - 4x_3 = -1 \end{array} \right\} \Rightarrow \left[\begin{array}{ccc|c} 1 & -4 & 7 & -6 \\ -5 & 1 & 2 & 11 \\ 2 & 3 & -4 & -1 \end{array} \right]$$