

Exercise 4

Find the point of intersection of the lines $x_1 - 5x_2 = 1$ and $3x_1 - 7x_2 = 5$.

Solution

Solving the following system of equations will tell us the point at which the two lines intersect.

$$\begin{aligned}x_1 - 5x_2 &= 1 \\ 3x_1 - 7x_2 &= 5\end{aligned}$$

Write the implied augmented matrix of this system.

$$\left[\begin{array}{cc|c} 1 & -5 & 1 \\ 3 & -7 & 5 \end{array} \right]$$

To make the bottom left entry 0, multiply the first row by -3 and add it to the second row.

$$\left[\begin{array}{cc|c} 1 & -5 & 1 \\ 0 & 8 & 2 \end{array} \right]$$

Divide the second row by 8.

$$\left[\begin{array}{cc|c} 1 & -5 & 1 \\ 0 & 1 & \frac{1}{4} \end{array} \right]$$

Now that the augmented matrix is in triangular form, write the corresponding system of equations.

$$\begin{aligned}x_1 - 5x_2 &= 1 \\ x_2 &= \frac{1}{4}\end{aligned}$$

Since x_2 is known now, x_1 can be found.

$$x_1 - 5\left(\frac{1}{4}\right) = 1 \quad \rightarrow \quad x_1 = \frac{9}{4}$$

Therefore, $x_1 = 9/4$ and $x_2 = 1/4$.