Exercise 5

Consider each matrix in Exercises 5 and 6 as the augmented matrix of a linear system. State in words the next two elementary row operations that should be performed in the process of solving the system.

$$\begin{bmatrix} 1 & -4 & 5 & 0 & 7 \\ 0 & 1 & -3 & 0 & 6 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -5 \end{bmatrix}$$

Solution

Multiply the third row by 3 and add it to the second row.

$$\left[\begin{array}{cccc|c} 1 & -4 & 5 & 0 & 7 \\ 0 & 1 & 0 & 0 & 12 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -5 \end{array}\right]$$

Multiply the third row by -5 and add it to the first row.

$$\left[\begin{array}{cccc|c} 1 & -4 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 & 12 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -5 \end{array}\right]$$

Multiply the second row by 4 and add it to the first row.

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 45 \\ 0 & 1 & 0 & 0 & 12 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -5 \end{array}\right]$$

The solution to the system of equations is now known.

$$\begin{aligned} x_1 &= 45 \\ x_2 &= 12 \\ x_3 &= 2 \\ x_4 &= -5 \end{aligned}$$