

**Exercise 6**

Consider each matrix in Exercises 5 and 6 as the augmented matrix of a linear system. State in words the next two elementary row operations that should be performed in the process of solving the system.

$$\left[ \begin{array}{cccc|c} 1 & -6 & 4 & 0 & -1 \\ 0 & 2 & -7 & 0 & 4 \\ 0 & 0 & 1 & 2 & -3 \\ 0 & 0 & 3 & 1 & 6 \end{array} \right]$$

**Solution**

Multiply the third row by  $-3$  and add it to the fourth row.

$$\left[ \begin{array}{cccc|c} 1 & -6 & 4 & 0 & -1 \\ 0 & 2 & -7 & 0 & 4 \\ 0 & 0 & 1 & 2 & -3 \\ 0 & 0 & 0 & -5 & 15 \end{array} \right]$$

Divide the fourth row by  $-5$ .

$$\left[ \begin{array}{cccc|c} 1 & -6 & 4 & 0 & -1 \\ 0 & 2 & -7 & 0 & 4 \\ 0 & 0 & 1 & 2 & -3 \\ 0 & 0 & 0 & 1 & -3 \end{array} \right]$$

Multiply the fourth row by  $-2$  and add it to the third row.

$$\left[ \begin{array}{cccc|c} 1 & -6 & 4 & 0 & -1 \\ 0 & 2 & -7 & 0 & 4 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & -3 \end{array} \right]$$

Multiply the third row by  $7$  and add it to the second row.

$$\left[ \begin{array}{cccc|c} 1 & -6 & 4 & 0 & -1 \\ 0 & 2 & 0 & 0 & 25 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & -3 \end{array} \right]$$

Divide the second row by  $2$ .

$$\left[ \begin{array}{cccc|c} 1 & -6 & 4 & 0 & -1 \\ 0 & 1 & 0 & 0 & \frac{25}{2} \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & -3 \end{array} \right]$$

Multiply the second row by  $6$  and add it to the first row.

$$\left[ \begin{array}{cccc|c} 1 & 0 & 4 & 0 & 74 \\ 0 & 1 & 0 & 0 & \frac{25}{2} \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & -3 \end{array} \right]$$

Multiply the third row by  $-4$  and add it to the first row.

$$\left[ \begin{array}{cccc|c} 1 & 0 & 0 & 0 & 62 \\ 0 & 1 & 0 & 0 & \frac{25}{2} \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & -3 \end{array} \right]$$

The solution to the system of equations is now known.

$$x_1 = 62$$

$$x_2 = \frac{25}{2}$$

$$x_3 = 3$$

$$x_4 = -3$$