

Exercise 9

In Exercises 7–10, the augmented matrix of a linear system has been reduced by row operations to the form shown. In each case, continue the appropriate row operations and describe the solution set of the original system.

$$\left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & -4 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 2 & 4 \end{array} \right]$$

Solution

Divide the fourth row by 2.

$$\left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & -4 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 1 & 2 \end{array} \right]$$

Multiply the fourth row by 3 and add it to the third row.

$$\left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & -4 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 2 \end{array} \right]$$

Multiply the third row by 3 and add it to the second row.

$$\left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 0 & 8 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 2 \end{array} \right]$$

Add the second row to the first row.

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 & 8 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 2 \end{array} \right]$$

Therefore,

$$x_1 = 4$$

$$x_2 = 8$$

$$x_3 = 5$$

$$x_4 = 2$$

and the solution set is $\{4, 8, 5, 2\}$.