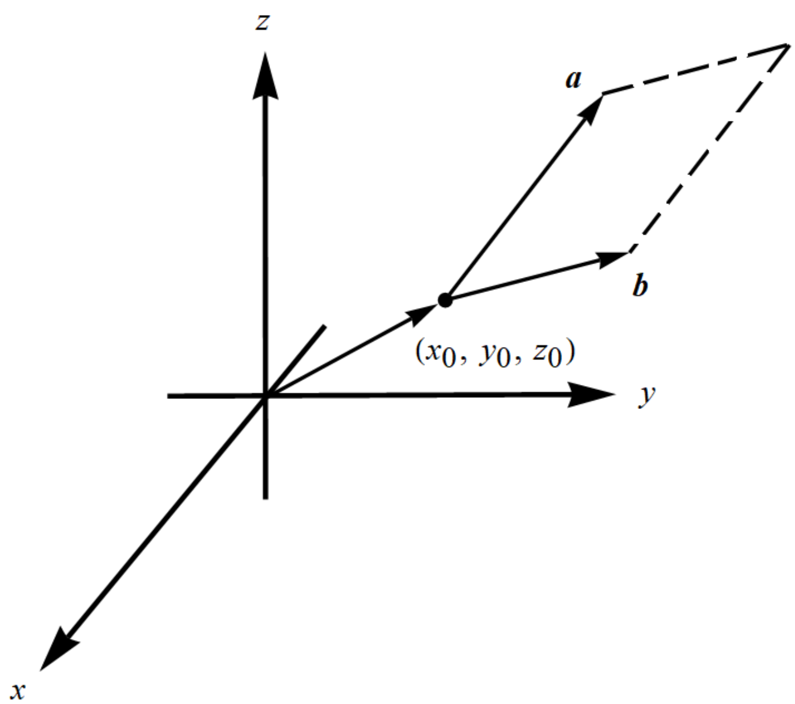
Exercise 30

In Exercises 29 to 31, use vector methods to describe the given configurations.

> The points within the parallelogram with one corner at (x_0, y_0, z_0) whose sides extending from that corner are equal in magnitude and direction to vectors $\mathbf{a}$ and $\mathbf{b}$

Solution



Assuming that $\mathbf{a}$ and $\mathbf{b}$ are linearly independent, a linear combination of these two spans an entire plane in three-dimensional space.

$$\mathbf{r}(s,t) = s\mathbf{a} + t\mathbf{b}$$

By restricting s and t to be between 0 and 1, only the points within the parallelogram with edge vectors, $\mathbf{a}$ and $\mathbf{b}$, are obtained. One of this parallelogram's corners is at the origin ($s = 0$ and $t = 0$). Adding the position vector (x_0, y_0, z_0) to $\mathbf{r}(s,t)$ makes it so that this corner is at (x_0, y_0, z_0) instead.

$$\{(x_0, y_0, z_0) + s\mathbf{a} + t\mathbf{b},\ 0 \le s \le 1,\ 0 \le t \le 1\}$$