

Exercise 15

What is the geometric relation between the vectors \mathbf{v} and \mathbf{w} if $\mathbf{v} \cdot \mathbf{w} = -\|\mathbf{v}\|\|\mathbf{w}\|$?

Solution

Note that the dot product of two vectors, \mathbf{v} and \mathbf{w} , is defined as

$$\mathbf{v} \cdot \mathbf{w} = \|\mathbf{v}\|\|\mathbf{w}\| \cos \theta,$$

where θ is the angle between the vectors. If $\mathbf{v} \cdot \mathbf{w} = -\|\mathbf{v}\|\|\mathbf{w}\|$, then

$$\cos \theta = -1,$$

which means $\theta = \pi$. The two vectors therefore have the same direction but opposite sense; that is, they are antiparallel.