Exercise 21

Find the projection of $\mathbf{v} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$ onto $\mathbf{u} = -\mathbf{i} + \mathbf{j} + \mathbf{k}$.

Solution

The dot product of $\mathbf{v}$ and $\hat{\mathbf{u}}$ represents the component of $\mathbf{v}$ in the direction of $\mathbf{u}$.

$$\begin{aligned}
\mathbf{v} \cdot \hat{\mathbf{u}} &= \mathbf{v} \cdot \frac{\mathbf{u}}{\|\mathbf{u}\|} \\
&= \frac{(2\mathbf{i} + \mathbf{j} - 3\mathbf{k}) \cdot (-\mathbf{i} + \mathbf{j} + \mathbf{k})}{\sqrt{(-1)^2 + 1^2 + 1^2}} \\
&= \frac{(2)(-1) + (1)(1) + (-3)(1)}{\sqrt{3}} \\
&= -\frac{4}{\sqrt{3}}
\end{aligned}$$

Multiply this result by a unit vector in the direction of $\mathbf{u}$ to obtain the desired projection.

$$\begin{aligned}
(\mathbf{v} \cdot \hat{\mathbf{u}})\hat{\mathbf{u}} &= (\mathbf{v} \cdot \hat{\mathbf{u}})\frac{\mathbf{u}}{\|\mathbf{u}\|} \\
&= \left(-\frac{4}{\sqrt{3}}\right) \frac{-\mathbf{i} + \mathbf{j} + \mathbf{k}}{\sqrt{(-1)^2 + (1)^2 + (1)^2}} \\
&= \left(-\frac{4}{\sqrt{3}}\right) \frac{-\mathbf{i} + \mathbf{j} + \mathbf{k}}{\sqrt{3}} \\
&= -\frac{4}{3}(-\mathbf{i} + \mathbf{j} + \mathbf{k})
\end{aligned}$$