Exercise 31

An airplane is located at position $(3, 4, 5)$ at noon and traveling with velocity $400\mathbf{i} + 500\mathbf{j} - \mathbf{k}$ kilometers per hour. The pilot spots an airport at position $(23, 29, 0)$.

(a) At what time will the plane pass directly over the airport? (Assume that the plane is flying over flat ground and that the vector $\mathbf{k}$ points straight up.)

(b) How high above the airport will the plane be when it passes?

Solution

The airplane's position at time t (in hours) is

$$\mathbf{x}(t) = \mathbf{v}t + \mathbf{x}_0 \text{ kilometers,}$$

where $\mathbf{v}$ is the constant velocity vector and $\mathbf{x}_0$ is the airplane's initial position vector.

$$\begin{aligned}\mathbf{x}(t) &= (400, 500, -1)t + (3, 4, 5)\\ &= (400t, 500t, -t) + (3, 4, 5)\\ &= (400t + 3, 500t + 4, -t + 5)\end{aligned}$$

Part (a)

Set the airplane's position equal to that of the airport

$$(400t + 3, 500t + 4, -t + 5) = (23, 29, 0)$$

and match the first two components.

$$\left.\begin{aligned}400t + 3 = 23\\ 500t + 4 = 29\end{aligned}\right\} \quad \rightarrow \quad t = \frac{1}{20}\,\cancel{\text{hours}} \times \frac{60 \text{ minutes}}{1\,\cancel{\text{hour}}} = 3 \text{ minutes}$$

The plane will fly over the airport at 12:03 P.M.

Part (b)

Set $t = 1/20$ hours in the formula for the airplane's position.

$$\begin{aligned}\mathbf{x}\left(\frac{1}{20}\right) &= \left(400 \cdot \frac{1}{20} + 3, 500 \cdot \frac{1}{20} + 4, -\frac{1}{20} + 5\right)\\ &= \left(23, 29, \frac{99}{20}\right) \text{ kilometers}\end{aligned}$$

The plane's height above the ground is the third component,

$$\frac{99}{20} \text{ kilometers} = 4.95 \text{ kilometers.}$$