Exercise 32

Find a parametrization for the line perpendicular to $(2, -1, 1)$, parallel to the plane $2x + y - 4z = 1$, and passing through the point $(1, 0, -3)$.

[TYPO: There's a period missing at the end.]

Solution

The equation for a line is

$$\mathbf{y}(t) = \mathbf{m}t + \mathbf{b},$$

where $\mathbf{m}$ is the direction vector and $\mathbf{b}$ is the position vector for any point the line goes through. The direction vector is perpendicular to both $(2, -1, 1)$ and the normal vector of the plane, $(2, 1, -4)$, which is obtained from the coefficients of x, y, and z. Take the cross product of these two vectors to get $\mathbf{m}$.

$$\mathbf{m} = (2, -1, 1) \times (2, 1, -4) = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ 2 & -1 & 1 \\ 2 & 1 & -4 \end{vmatrix} = (4-1)\hat{\mathbf{x}} - (-8-2)\hat{\mathbf{y}} + (2+2)\hat{\mathbf{z}} = 3\hat{\mathbf{x}} + 10\hat{\mathbf{y}} + 4\hat{\mathbf{z}} = (3, 10, 4)$$

The position vector for a point the line passes through is $(1, 0, -3)$.

$$\begin{aligned} \mathbf{y}(t) &= (3, 10, 4)t + (1, 0, -3) \\ &= (3t, 10t, 4t) + (1, 0, -3) \\ &= (3t + 1, 10t, 4t - 3) \end{aligned}$$