Exercise 35

Find an equation for the plane that contains the line $\mathbf{v} = (-1, 1, 2) + t(3, 2, 4)$ and is perpendicular to the plane $2x + y - 3z + 4 = 0$.

Solution

The equation for a plane is

$$\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0,$$

where $\mathbf{n}$ is a vector normal to the plane and $\mathbf{r}_0$ is the position vector for any point in the plane. The normal vector is perpendicular to both the direction the line goes in, $(3, 2, 4)$, and the normal vector of the given plane, $(2, 1, -3)$. Take the cross product of these two to obtain it.

$$\mathbf{n} = (3,2,4)\times(2,1,-3) = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ 3 & 2 & 4 \\ 2 & 1 & -3 \end{vmatrix} = (-6-4)\hat{\mathbf{x}} - (-9-8)\hat{\mathbf{y}} + (3-4)\hat{\mathbf{z}} = -10\hat{\mathbf{x}} + 17\hat{\mathbf{y}} - \hat{\mathbf{z}} = (-10, 17, -1)$$

Set $t = 0$ to get the position vector for a point on the line. This will be $\mathbf{r}_0$: $\mathbf{r}_0 = (-1, 1, 2)$.

$$(-10, 17, -1) \cdot (x + 1, y - 1, z - 2) = 0$$

$$-10(x + 1) + 17(y - 1) - 1(z - 2) = 0$$

$$-10x - 10 + 17y - 17 - z + 2 = 0$$

$$-10x + 17y - z = 25$$

$$10x - 17y + z = -25$$