Exercise 37

Redo Exercises 25 and 26 of Section 1.1 using the dot product and what you know about normals to planes.

Solution

Exercise 25

Show that there are no points (x, y, z) satisfying $2x - 3y + z - 2 = 0$ and lying on the line $\mathbf{v} = (2, -2, -1) + t(1, 1, 1)$.

The vector normal to the plane is obtained from the coefficients of x, y, and z: $(2, -3, 1)$. Take the dot product of it and the direction vector of the line.

$$(2, -3, 1) \cdot (1, 1, 1) = 2 - 3 + 1 = 0$$

Because it's zero, the plane is parallel with the line. Setting $t = 0$ gives $(2, -2, -1)$, a point that the line goes through. Plug in $x = 2$, $y = -2$, and $z = -1$ to the equation of the plane.

$$2(2) - 3(-2) + (-1) - 2 = 4 + 6 - 1 - 2 = 7 \neq 0$$

Because it's not zero, the point $(2, -2, -1)$ and every other on the line do not lie in the plane.

Exercise 26

Show that every point on the line $\mathbf{v} = (1, -1, 2) + t(2, 3, 1)$ satisfies the equation $5x - 3y - z - 6 = 0$.

The vector normal to the plane is obtained from the coefficients of x, y, and z: $(5, -3, -1)$. Take the dot product of it and the direction vector of the line.

$$(5, -3, -1) \cdot (2, 3, 1) = 10 - 9 - 1 = 0$$

Because it's zero, the plane is parallel with the line. Setting $t = 0$ gives $(1, -1, 2)$, a point that the line goes through. Plug in $x = 1$, $y = -1$, and $z = 2$ to the equation of the plane.

$$5(1) - 3(-1) - (2) - 6 = 5 + 3 - 2 - 6 = 0$$

Because it's zero, the point $(1, -1, 2)$ and every other on the line lie in the plane.