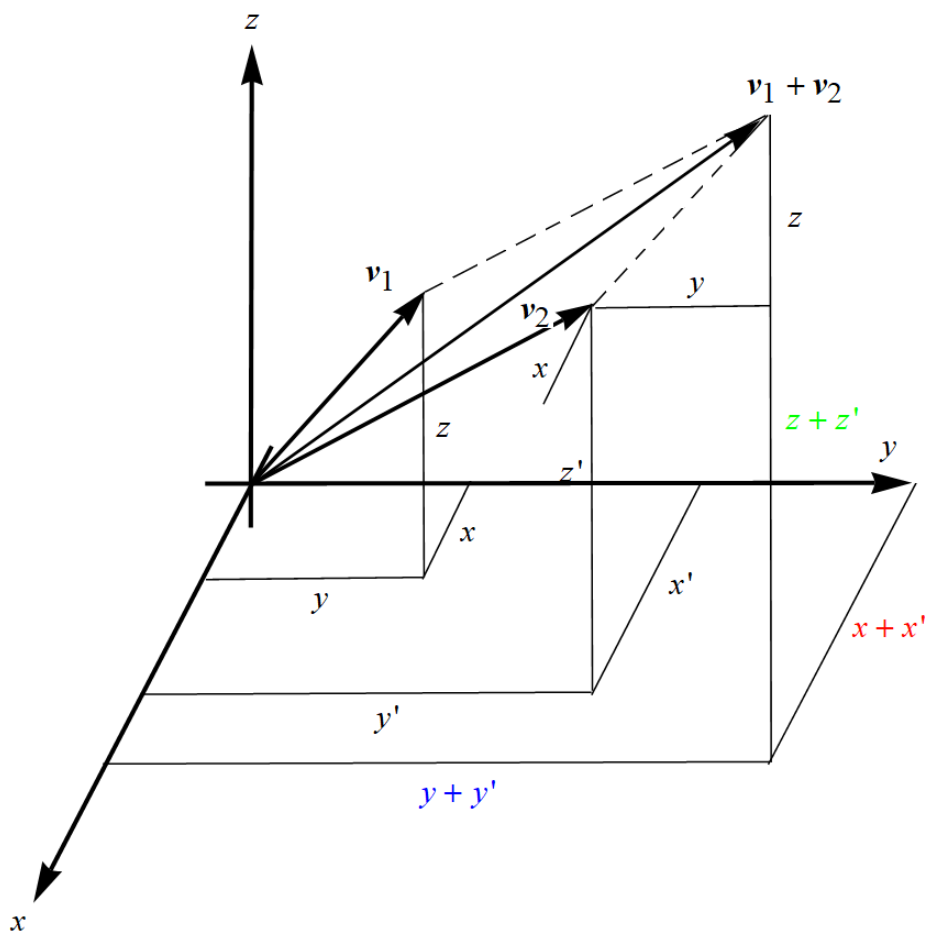
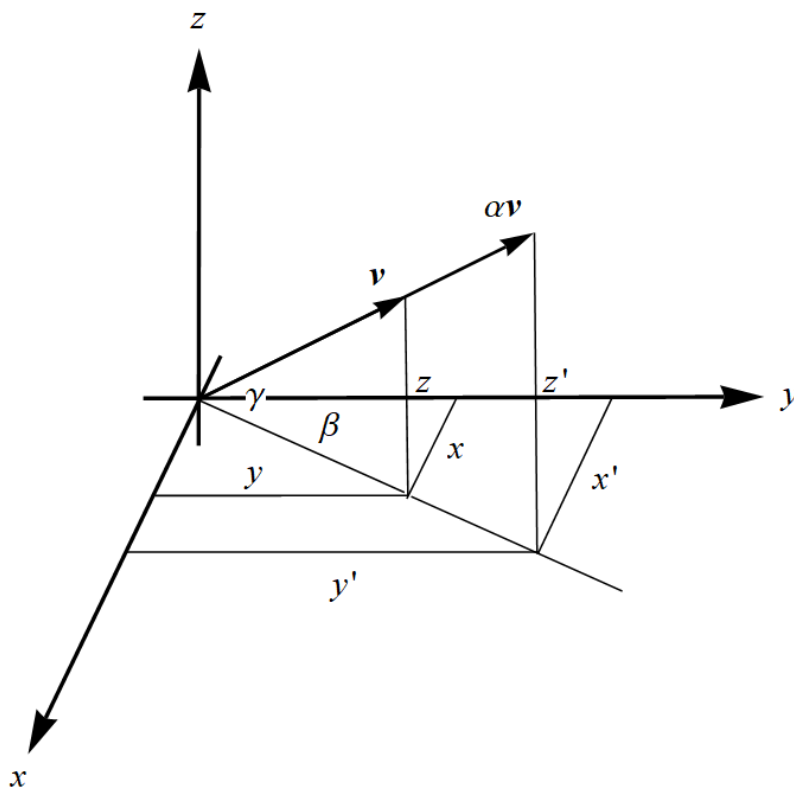


Exercise 12

- (a) Generalize the geometric construction in Figure 1.1.7 to show that if $\mathbf{v}_1 = (x, y, z)$ and $\mathbf{v}_2 = (x', y', z')$, then $\mathbf{v}_1 + \mathbf{v}_2 = (x + x', y + y', z + z')$.
- (b) Using an argument based on similar triangles, prove that $\alpha\mathbf{v} = (\alpha x, \alpha y, \alpha z)$ when $\mathbf{v} = (x, y, z)$.
-

Solution**Part (a)**

Part (b)

Multiplying a vector by a number α changes its magnitude by a factor of α as shown in the figure above. By the Pythagorean theorem, the hypotenuses in the xy -plane have lengths, $\sqrt{x^2 + y^2}$ and $\sqrt{x'^2 + y'^2}$, respectively.

$$\sin \gamma = \frac{z}{|\mathbf{v}|} = \frac{z'}{|\alpha \mathbf{v}|} \rightarrow \frac{z}{\sqrt{x^2 + y^2 + z^2}} = \frac{z'}{\alpha \sqrt{x^2 + y^2 + z^2}} \rightarrow z' = \alpha z$$

$$\tan \gamma = \frac{z}{\sqrt{x^2 + y^2}} = \frac{z'}{\sqrt{x'^2 + y'^2}} \rightarrow \frac{\sqrt{x'^2 + y'^2}}{\sqrt{x^2 + y^2}} = \frac{z'}{z} = \alpha$$

$$\cos \beta = \frac{y}{\sqrt{x^2 + y^2}} = \frac{y'}{\sqrt{x'^2 + y'^2}} \rightarrow \frac{\sqrt{x'^2 + y'^2}}{\sqrt{x^2 + y^2}} = \alpha = \frac{y'}{y} \rightarrow y' = \alpha y$$

$$\sin \beta = \frac{x}{\sqrt{x^2 + y^2}} = \frac{x'}{\sqrt{x'^2 + y'^2}} \rightarrow \frac{\sqrt{x'^2 + y'^2}}{\sqrt{x^2 + y^2}} = \alpha = \frac{x'}{x} \rightarrow x' = \alpha x$$

Therefore, $\alpha \mathbf{v} = (x', y', z') = (\alpha x, \alpha y, \alpha z)$.