

Exercise 20

Show that $\mathbf{l}_1(t) = (1, 2, 3) + t(1, 0, -2)$ and $\mathbf{l}_2(t) = (2, 2, 1) + t(-2, 0, 4)$ parametrize the same line.

Solution

The equation for a line is

$$\mathbf{y}(t) = \mathbf{m}t + \mathbf{b},$$

where \mathbf{m} is the direction vector, \mathbf{b} is the position vector for a point on the line, and t is a parameter. \mathbf{l}_1 is parallel with \mathbf{l}_2 because its direction vector is a constant multiple of that for \mathbf{l}_2 : $-2(1, 0, -2) = (-2, 0, 4)$. Since $\mathbf{l}_1(-1) = \mathbf{l}_2(1) = (0, 2, 5)$, the two lines travel through the same point. $\mathbf{l}_1(t)$ and $\mathbf{l}_2(t)$ are therefore just two different ways of parameterizing the same line.