

## Exercise 21

Do the points  $(2, 3, -4)$ ,  $(2, 1, -1)$ , and  $(2, 7, -10)$  lie on the same line?

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### Solution

The equation for a line is

$$\mathbf{y}(t) = \mathbf{m}t + \mathbf{b},$$

where  $\mathbf{m}$  is the direction vector,  $\mathbf{b}$  is the position vector for a point on the line, and  $t$  is a parameter. Subtract any two of the given position vectors to get the direction vector.

$$\begin{aligned}\mathbf{m} &= (2, 3, -4) - (2, 1, -1) \\ &= (0, 2, -3)\end{aligned}$$

So then an equation for the line in question is

$$\begin{aligned}\mathbf{y}(t) &= (0, 2, -3)t + (2, 3, -4) \\ &= (0, 2t, -3t) + (2, 3, -4) \\ &= (2, 2t + 3, -3t - 4)\end{aligned}$$

Set  $t = 0$  to get the first point.

$$\mathbf{y}(0) = (2, 3, -4)$$

Set  $t = -1$  to get the second point.

$$\mathbf{y}(-1) = (2, 1, -1)$$

Set  $t = 2$  to get the third point.

$$\mathbf{y}(2) = (2, 7, -10)$$

Therefore, all three points lie on the same line.