Exercise 22

Let $\mathbf{u} = (1, 2)$, $\mathbf{v} = (-3, 4)$, and $\mathbf{w} = (5, 0)$:

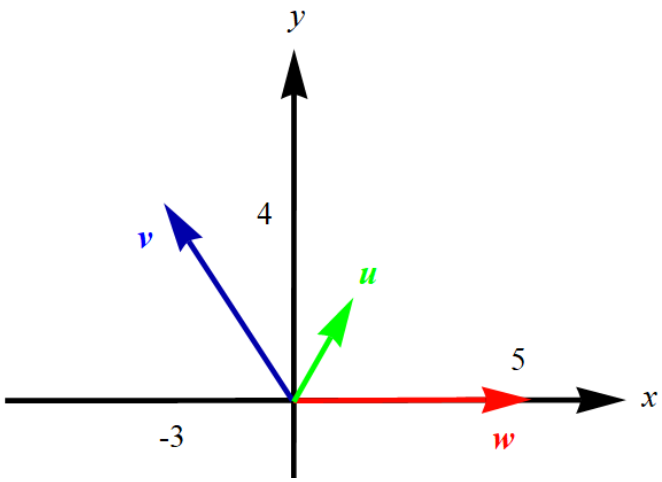
(a) Draw these vectors in $\mathbb{R}^2$.

(b) Find scalars λ_1 and λ_2 such that $\mathbf{w} = \lambda_1\mathbf{u} + \lambda_2\mathbf{v}$.

Solution

Part (a)

The three vectors are illustrated below.



Part (b)

The aim is to find scalars, λ_1 and λ_2, that satisfy $\mathbf{w} = \lambda_1\mathbf{u} + \lambda_2\mathbf{v}$.

$$\begin{aligned}(5, 0) &= \lambda_1(1, 2) + \lambda_2(-3, 4) \\ &= (\lambda_1, 2\lambda_1) + (-3\lambda_2, 4\lambda_2) \\ &= (\lambda_1 - 3\lambda_2, 2\lambda_1 + 4\lambda_2)\end{aligned}$$

The respective components of each vector are equal.

$$\begin{aligned}5 &= \lambda_1 - 3\lambda_2 \\ 0 &= 2\lambda_1 + 4\lambda_2\end{aligned}$$

Solving this system of equations yields $\lambda_1 = 2$ and $\lambda_2 = -1$. Therefore, $\mathbf{w} = 2\mathbf{u} - \mathbf{v}$.