

**Exercise 26**

Show that every point on the line  $\mathbf{v} = (1, -1, 2) + t(2, 3, 1)$  satisfies the equation  $5x - 3y - z - 6 = 0$ .

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**Solution**

The parameterization for the line can be written as

$$\begin{aligned}\mathbf{v} &= (1, -1, 2) + t(2, 3, 1) \\ &= (1, -1, 2) + (2t, 3t, t) \\ &= (1 + 2t, -1 + 3t, 2 + t).\end{aligned}$$

The  $x$ -,  $y$ -, and  $z$ -components of the line are

$$x = 1 + 2t \quad \text{and} \quad y = -1 + 3t \quad \text{and} \quad z = 2 + t,$$

respectively. Substitute these into the equation for the plane.

$$\begin{aligned}5x - 3y - z - 6 &= 5(1 + 2t) - 3(-1 + 3t) - (2 + t) - 6 \\ &= 5 + 10t + 3 - 9t - 2 - t - 6 \\ &= 0\end{aligned}$$

Since the right side is 0 for all values of  $t$ , every point  $(x, y, z)$  on the line lies in the plane.